Worksheet 2.2 Solving Equations in One Variable

Section 1 Simple Examples

You are on your way to Brisbane from Sydney, and you know that the trip is 1100 km. You pass a sign that says that Brisbane is now 200 km away. How far have you travelled? We can see the answer quickly, but we shall write the information down in the form of a mathematical expression. Let x be the distance travelled so far. Then

$$x + 200 = 1100$$

This is the information that we are given. To find the answer we are to solve an equation in one variable, x.

$$\begin{aligned} x + 200 &= 1100 \\ x + 200 - 200 &= 1100 - 200 \qquad \text{subtract 200 from both sides} \\ x &= 900 \end{aligned}$$

The middle step emphasizes what we do in the calculation. We wanted x by itself on one side of the equation, so we subtracted 200 from each side of the equation. An equation is like a balancing beam in the sense that if you do anything to one side of the equation, you must also do it to the other.

Example 1 : If $x + 2 = 3$, what is x? Clearly, $x = 1$, but again we follow through the calculation.

$$\begin{aligned} x + 2 &= 3 \\ x + \cancel{2} - \cancel{2} &= 3 - 2 \\ x &= 1 \end{aligned}$$

Example 2 : If $x - 5 = -12$, what is x?

$$\begin{aligned} x - \cancel{5} + \cancel{5} &= -12 + 5 \qquad \text{adding 5 to both sides} \\ x &= -7 \end{aligned}$$

Example 3 : Solve for x in $-8 = -10 + x$.

$$\begin{aligned} -8 &= -10 + x \\ -8 + 10 &= -10 + x + 10 \\ x &= 2 \end{aligned}$$

In each of the above calculations, we worked on the equation by trying to get x on one side and all the rest of the expression on the other side. This is a basic method for solving equations in one variable. To simplify the expression involving x we simply undid whatever was done to x by applying the opposite arithmetic operation to leave us with just an x on one side. *Most importantly*, we need to remember that whatever we do to one side of the equation, we must do to the other. For example, if we subtract 5 from one side of the equation, we must subtract 5 from the other.

Exercises:

1. Solve the following equations:

   (a) $x + 9 = 20$

   (b) $x - 8 = 10$

   (c) $x + 1.6 = 2.4$

   (d) $x - \frac{3}{4} = 1\frac{1}{2}$

   (e) $y - 2 = -8$

   (f) $m + 6 = -4$

   (g) $m - 7.1 = -8.4$

   (h) $t - 2.4 = -1$

   (i) $x + 1\frac{1}{2} = 2$

   (j) $y + 80 = 120$

Section 2 Equations Involving Multiplication

In a similar way to the above we can deal with solving equations such as

$$5x = 2$$

The arithmetic operation is now multiplication, whose inverse is division. The inverse of multiplying by 5 is dividing by 5.

$$\begin{aligned} 5x &= 2 \quad \text{becomes} \\ \frac{\cancel{5}}{\cancel{5}}x &= \frac{2}{5} \quad \text{dividing both sides by 5} \\ 1x &= \frac{2}{5} \\ x &= \frac{2}{5} \end{aligned}$$

Example 1 :

$$\begin{aligned} \frac{x}{3} &= 2 \quad \text{becomes} \\ \frac{x}{\not{3}} \times \not{3} &= 2 \times 3 \quad \text{multiplying both sides by 3} \\ x &= 6 \end{aligned}$$

Example 2 :

$$\begin{aligned} \frac{2}{7}x &= 4 \\ \not{7} \times \frac{2}{7}x &= 4 \times 7 \qquad \text{(multiplying both sides by 7)} \\ 2x &= 28 \\ \frac{\not{2}x}{\not{2}} &= \frac{28}{2} \qquad \text{(dividing both sides by 2)} \\ x &= 14 \end{aligned}$$

Exercises:

1. Solve the following equations:

   (a) $4x = 20$

   (b) $6x = 24$

   (c) $\frac{x}{3} = 5$

   (d) $\frac{x}{5} = 1.2$

   (e) $2x = 11$

   (f) $3x = -24$

   (g) $\frac{x}{2} = -7$

   (h) $\frac{3x}{4} = -12$

   (i) $\frac{2}{5}m = 10$

   (j) $\frac{3m}{4} = -6$

Section 3 Multiple Terms

The puzzle page in the newspapers sometimes has puzzles like this example:

Example 1 : Jean is 7 years older than half of Tom's age. If Jean is 35, how old is Tom? We now write the information that we are given in the form of an algebraic expression. Let Jean's age be J and Tom's T. Then

$$J = \frac{1}{2}T + 7$$

Putting in J as 35, we get

$$35 = \frac{1}{2}T + 7$$

What is T? Notice that we now have a combination of arithmetic operations to deal with. It doesn't matter what order you do them in so long as you remember to include the whole of each side of the equation in the undoing, and whatever you do to one side you must do the same to the other side. We will solve for T in two ways.

Solution 1

$$\begin{aligned} 35 &= \frac{1}{2}T + 7 \\ 35 - 7 &= \frac{1}{2}T + 7 - 7 \quad \text{(subtract 7 from both sides)} \\ 28 &= \frac{1}{2}T \\ 2 \times 28 &= 2 \times \frac{1}{2}T \quad \text{(multiply both sides by 2)} \\ 56 &= T \end{aligned}$$

Solution 2

$$\begin{aligned} 35 &= \frac{1}{2}T + 7 \\ 2 \times 35 &= 2 \times (\frac{1}{2}T + 7) \quad \text{(multiply both \underline{sides} by 2)} \\ 70 &= T + 2 \times 7 \quad \text{(multiply out the brackets)} \\ 70 &= T + 14 \\ 70 - 14 &= T + 14 - 14 \\ 56 &= T \end{aligned}$$

<u>Note:</u> When multiplying through by a number it is important to multiply every term on both sides of the equation. On the whole, it is probably easier to undo addition and subtraction first.

<u>Example 2</u> :

$$\begin{aligned} 3x - 5 &= 16 \\ 3x - 5 + 5 &= 16 + 5 \qquad \text{(add 5 to both sides)} \\ 3x &= 21 \\ \frac{3x}{3} &= \frac{21}{3} \\ x &= 7 \end{aligned}$$

Occasionally you may be asked to solve an equation in one variable where the variable appears on both sides of the equation.

Example 3 : Solve

$$3x + 5 = 2x + 1 \qquad \text{for } x$$

We will undo the operations in a way that puts all the x parts of the expression on one side of the equation.

$$\begin{aligned}
3x + 5 &= 2x + 1 \\
3x + 5 - 2x &= 2x + 1 - 2x \qquad \text{(subtract } 2x \text{ from both sides)} \\
x + 5 &= 1 \\
x + 5 - 5 &= 1 - 5 \qquad \text{(subtract 5 from both sides)} \\
x &= -4
\end{aligned}$$

Sometimes the expressions will be more complex and could involve brackets. In these cases we expand out the brackets and proceed.

Example 4 :

$$\begin{aligned}
3(5 - x) - 2(5 + x) &= 3(x + 1) \\
15 - 3x - 10 - 2x &= 3x + 3 \qquad \text{(collect like terms)} \\
5 - 5x &= 3x + 3 \\
5 - 5x + 5x &= 3x + 3 + 5x \qquad \text{(add } 5x \text{ to both sides)} \\
5 &= 8x + 3 \\
5 - 3 &= 8x + 3 - 3 \qquad \text{(subtract 3 from both sides)} \\
2 &= 8x \\
\frac{2}{8} &= \frac{8x}{8} \qquad \text{(divide both sides by 8)} \\
\frac{1}{4} &= x
\end{aligned}$$

Once you feel confident in these processes there is no need to put in the intermediate steps illustrated in these examples.

Exercises:

1. Solve the following equations:

   (a) $2x - 1 = 9$

   (b) $\frac{y}{3} + 4 = 12$

   (c) $2(x + 1) - 7 = 5$

   (d) $4(y + 3) - 2y = 7$

   (e) $5(y + 2) - 4(y - 1) = 6$

   (f) $5(2 - x) - 3(4 - 2x) = 20$

   (g) $2m + 4 - 3m = 8(m - 1)$

   (h) $3m + 12 = 2(m - 3) + 4$

   (i) $\frac{x+1}{4} = 5$

   (j) $\frac{x}{5} + \frac{x}{3} = 10$

Exercises 2.2 Solving Equations in One Variable

1. Solve

   (a) $x + 4 = -7$

   (b) $2 - x = 13$

   (c) $15y = 45$

   (d) $-\frac{t}{2} = -9$

   (e) $3y - 20 = \frac{1}{2}$

   (f) $\frac{x+3}{2} = -1$

   (g) $3x + 2 = 4x - 7$

   (h) $\frac{x}{2} + 7 = \frac{3x}{4}$

   (i) $2x(x + 3) = 2x^2 + 15$

   (j) $(y + 7)(y + 7) = y^2$

   (k) $2x + 7 + 8x = 13$

   (l) $3(x + 1) + 4x = 26$

   (m) $8(m - 3) - 2(m - 2) = 20$

   (n) $\frac{y+3}{2} = \frac{y-4}{3}$

   (o) $3(4 - y) = 2(y + 5)$

   (p) $\frac{x}{7} = 3\frac{1}{7}$

   (q) $\frac{x+1}{2} = \frac{3}{4}$

   (r) $16t - 7 + 4t = 12t - 1$

   (s) $\frac{t}{4} + 3 = \frac{t}{8} - 1$

   (t) $8 = \frac{1}{3}T + 2$

2. (a) Three times a number is equal to the number decreased by two. What is the number?

   (b) The sum of two consecutive numbers is 93. What are the numbers?

   (c) The sum of two consecutive *even* numbers is 46. Find the numbers.

   (d) When the tax on cigarettes was increased by 5%, the price of a certain pack became \$5.60. What was the oringinal price?

   (e) In 1994, 15% of the women who went on maternity leave returned to full-time work, while in 1993 only 12% returned. If the number returning in each year was 6, how many left to have babies in 1994?

   (f) Two times a number is equal to six less than three times the number. What is the number?

Answers 2.2

Section 1

1. (a) 11
   (b) 18
   (c) 0.8
   (d) $2\frac{1}{4}$
   (e) -6
   (f) -10
   (g) -1.3
   (h) 1.4
   (i) $\frac{1}{2}$
   (j) 40

Section 2

1. (a) 5
   (b) 4
   (c) 15
   (d) 6
   (e) $5\frac{1}{2}$
   (f) -8
   (g) -14
   (h) -16
   (i) 25
   (j) -8

Section 3

1. (a) 5
   (b) 24
   (c) 5
   (d) $-2\frac{1}{2}$
   (e) -8
   (f) 22
   (g) $1\frac{1}{3}$
   (h) -14
   (i) 19
   (j) $18\frac{3}{4}$

Exercises 2.2

1. (a) $x = -11$
   (b) $x = -11$
   (c) $y = 3$
   (d) $t = 18$
   (e) $y = \frac{41}{6}$
   (f) $x = -5$
   (g) $x = 9$
   (h) $x = 28$
   (i) $x = \frac{5}{2}$
   (j) $y = \frac{-7}{2}$
   (k) $x = \frac{3}{5}$
   (l) $x = 3\frac{2}{7}$
   (m) $m = 6\frac{2}{3}$
   (n) $y = -17$
   (o) $y = \frac{2}{5}$
   (p) $x = 22$
   (q) $x = \frac{1}{2}$
   (r) $t = \frac{3}{4}$
   (s) $t = -32$
   (t) $T = 18$

2. (a) -1
   (b) 46,47
   (c) 22,24
   (d) $5.33
   (e) 40
   (f) 6