I. A philosophical and historical perspective (cycles and elephants)

II. Local homeomorphisms and sections

III. Yoneda Lemma and Kan Theorem

IV. Presheaves, étale spaces, sheaves
   Examples: Sheaves of functions
   Spectrum and structure sheaf of a ring

V. Elementary toposes (development of intuitionistic set theory)
   Examples: Spatial toposes
   $G$-sets (or more generally $[G, Set]$)

VI. Geometric morphisms
   Examples: Generalized spaces (sobriety)
   Generalized groups
   Local structures (local-ringed spaces)

VII. Grothendieck topologies: lex totality; $\text{TopSp as a site}$

VIII. Classifying toposes

IX. Analysis in a topos

X. Representation theory of rings
   Example: Kaplansky becomes Swan

XI. Elementary set theory (axiom of choice and theorem of Barr)

XII. Non-standard analysis of A. Robinson

XIII. Independence results in set theory

XIV. Cohomology

XV. Stacks

XVI. Topological groupoids

XVII. Structure Theorem (Grothendieck–Toepel–Tierney)