Example II of Day–Street "Monoidal bicategories..."

We will take the antipode axiom at the bottom of page 142 in the form of a V-functor $S: H \to H^{op}$ together with a V-natural isomorphism

\[(*) \quad \mathcal{H}(B, C) \otimes \mathcal{H}(A, SB) \cong \mathcal{H}(B, C) \otimes \mathcal{H}(A, C).\]

Proposition Each Hopf monoid $H$ in a braided monoidal category $V$ gives a one-object example of a Hopf V-algebroid.

Proof Here $H$ has one object $A$ and $\mathcal{H}(A, A) = H$ so that $(*)$ becomes an isomorphism

\[H \otimes H \cong H \otimes H\]

for which we take the fusion map $N$. To see that it gives a $(*)$ which is natural, we must see what functionality of $(*)$ in $A, B, C$ means on the two sides. Both sides become left $H$, right $H \otimes H$-modules. The left $H \otimes H$ has action given by

\[\begin{array}{cccc}
H & \xrightarrow{H \otimes S \otimes H} & H \otimes \mathcal{H} & \xrightarrow{H \otimes S \otimes H} & H \otimes \mathcal{H} \\
H & \xrightarrow{H \otimes C_{H, H} \otimes H} & H \otimes \mathcal{H} & \xrightarrow{H \otimes C_{H, H} \otimes H} & H \otimes \mathcal{H}
\end{array}\]

i.e., $H \otimes \mathcal{H}$.

The right $H \otimes H$ has action given by

\[\begin{array}{cccc}
H \otimes \mathcal{H} & \xrightarrow{H \otimes S \otimes H} & H \otimes \mathcal{H} & \xrightarrow{H \otimes S \otimes H} & H \otimes \mathcal{H} \\
H \otimes \mathcal{H} & \xrightarrow{H \otimes C_{H, H} \otimes H} & H \otimes \mathcal{H} & \xrightarrow{H \otimes C_{H, H} \otimes H} & H \otimes \mathcal{H}
\end{array}\]

i.e., $H \otimes \mathcal{H}$.
The proof is completed by a string calculation using the Hoff manifold axioms:

\[ \text{Diagram} \]

as required. □