Making weak maps compose strictly

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Outline

Motivation

Weak maps of bicategories

Weak maps of tricategories

Weak maps of weak \(\omega\)-categories

(NB: Talk notes available at http://www.dpmms.cam.ac.uk/~rhgg2)
Consider a category \( \mathcal{C} \) with:

- **Objects** being higher-dimensional \( \cdots \);  
- **Morphisms** being strict structure-preserving maps.

Would like to derive \( \mathcal{C}_{\text{wk}} \) with:

- Same objects;  
- Morphisms being *weak* structure-preserving maps.
Motivation

Idea from homotopy theory: identify weak maps $X \to Y$ with strict maps $X' \to \tilde{Y}$ where:

- $X'$ is a cofibrant replacement for $X$;
- $\tilde{Y}$ is a fibrant replacement for $Y$. 
**Example: Ch(R)**

Ch(R), category of (positively graded) chain complexes over R.

- A strict map $X \to Y$ is a map of chain complexes;
Example: $\text{Ch}(R)$

$\text{Ch}(R)$, category of (positively graded) chain complexes over $R$.

- A strict map $X \rightarrow Y$ is a map of chain complexes;
- A strict map $X' \rightarrow Y$ is a map which preserves the $R$-module structure only up to homotopy;
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$\text{Ch}(R)$, category of (positively graded) chain complexes over $R$.

- A strict map $X \to Y$ is a map of chain complexes;
- A strict map $X' \to Y$ is a map which preserves the $R$-module structure only up to homotopy;
- A strict map $X \to \tilde{Y}$ is a map which preserves the differential only up to homotopy;
Ch(R), category of (positively graded) chain complexes over R.

- A strict map $X \to Y$ is a map of chain complexes;
- A strict map $X' \to Y$ is a map which preserves the $R$-module structure only up to homotopy;
- A strict map $X \to \tilde{Y}$ is a map which preserves the differential only up to homotopy;
- A strict map $X' \to \tilde{Y}$ (= weak map $X \to Y$) is a map which preserves the $R$-module structure and the differential only up to homotopy.
Example: $f/\text{Cat}/Z$

Let $f : X \to Z$ in $\text{Cat}$. Can form the interval category $f/\text{Cat}/Z$:

- **Objects** are $X \xrightarrow{g} Y \xrightarrow{h} Z$ with $hg = f$;
- **Morphisms** are commutative diamonds:

```
\begin{tikzcd}
X & Y \\
& W.
\arrow{g}{g} \arrow{h}{h} \arrow{y}{j} \arrow{w}{k}
\end{tikzcd}
```
Example: $f/C\text{at}/Z$

Corresponding *weak maps* should be pseudo-commutative diamonds:

\[
\begin{array}{c}
\text{Y} \\
\downarrow h \\
X \\
\downarrow j \\
\text{W} \\
\downarrow k \\
\text{Z}
\end{array}
\]

\[
\begin{array}{c}
g \\
\Rightarrow \\
j \\
\Rightarrow
\end{array}
\]
Example: $f/\text{Cat}/Z$

Corresponding *weak maps* should be pseudo-commutative diamonds:

![Diagram](https://via.placeholder.com/150)

... and these are precisely strict maps

![Diagram](https://via.placeholder.com/150)

where:
Example: $f$/Cat$/Z$

\[ \cdots \text{fibrant replacement of } X \xrightarrow{j} W \xrightarrow{k} Z \text{ is: } \]

\[ X \xrightarrow{\lambda_k \circ j} W \xrightarrow{\rho_k} Z \]
Example: $f/Cat/Z$

\[
\text{\ldots fibrant replacement of } X \xrightarrow{j} W \xrightarrow{k} Z \text{ is:} \\
X \xrightarrow{\lambda_k \circ j} W \xrightarrow{k} Z
\]

and cofibrant replacement of $X \xrightarrow{g} Y \xrightarrow{h} Z$ is:

\[
X \xrightarrow{l_g} g \xrightarrow{\approx} Y \xrightarrow{h \circ r_g} Z.
\]
How to compose weak maps?

Idea from category theory:

- Cofibrant replacement should be a comonad \((-)' : \mathcal{C} \to \mathcal{C}\);
- Fibrant replacement should be a monad \(\sim(-) : \mathcal{C} \to \mathcal{C}\);
- There should be a distributive law \(d_X : (\tilde{X})' \to \tilde{X}'\).
How to compose weak maps?

Idea from category theory:

- Cofibrant replacement should be a comonad \((-)’: \mathcal{C} \to \mathcal{C};\)
- Fibrant replacement should be a monad \(\sim(-): \mathcal{C} \to \mathcal{C};\)
- There should be a distributive law \(d_X: (\tilde{X})’ \to \tilde{X}'.\)

Now composition of weak maps is two-sided Kleisli composition:

\[
(X' \xrightarrow{f} \tilde{Y}) \circ (Y' \xrightarrow{g} \tilde{Z}) := X' \xrightarrow{\Delta_X} X'' \xrightarrow{f'} (\tilde{Y})' \xrightarrow{d_Y} \tilde{Y}' \xrightarrow{\tilde{g}} \tilde{Z} \xrightarrow{\mu_Z} \tilde{Z}.
\]
Example: $f/Cat/Z$

- Cofibrant replacement is a comonad [Grandis–Tholen 2006];
- Fibrant replacement is a monad [loc. cit.];
- There is a distributive law between them;

and corresponding Kleisli composition is what you think it is: pasting of pseudo-commutative diamonds.
In general

If a (locally presentable) category $\mathcal{C}$ has a cofibrantly generated model structure on it, then:

- Cofibrant replacement can be made a comonad [G. 2008];
- Fibrant replacement can be made a monad [loc. cit.];
- But not clear how to get a distributive law between them!

So in this talk, we focus on the case where every object is fibrant.
(As then we only need cofibrant replacement comonad).
Weak maps of bicategories

Consider the category $\text{Bicat}_s$:

- *Objects* are bicategories;
- *Morphisms* are strict homomorphisms.

There is a cofibrantly generated model structure on $\text{Bicat}_s$ [Lack, 2004], wherein:

- Weak equivalences are biequivalences;
- Every object is fibrant.

What are the corresponding weak maps?
First we describe cofibrant replacement comonad \((-)'\). It is generated by the following set of maps in $\text{Bicat}_3$:
Explicitly, if $\mathcal{B}$ is a bicategory, then $\mathcal{B}'$ is given as follows:

- Ignore the 2-cells and form the free bicategory $FUB$ on the underlying 1-graph of $\mathcal{B}$;
Explicitly, if $\mathcal{B}$ is a bicategory, then $\mathcal{B}'$ is given as follows:

- Ignore the 2-cells and form the free bicategory $FU\mathcal{B}$ on the underlying 1-graph of $\mathcal{B}$;
- Factorise the counit map $\varepsilon: FU\mathcal{B} \to \mathcal{B}$ as

$$FU\mathcal{B} \xrightarrow{a} \mathcal{B}' \xrightarrow{b} \mathcal{B}$$

where $a$ is bijective on objects and 1-cells and $b$ is locally fully faithful.

(NB: this is the flexible replacement of [Blackwell-Kelly-Power 1989]).
**Proposition (Coherence for homomorphisms)**

*The co-Kleisli category of \((-)' : \text{Bicat}_s \to \text{Bicat}_s* is isomorphic to the category \text{Bicat} of bicategories and homomorphisms.*
Proposition (Coherence for homomorphisms)

The co-Kleisli category of \((-)' : \text{Bicat}_s \to \text{Bicat}_s\) is isomorphic to the category \text{Bicat} of bicategories and homomorphisms.

Proof.

- First define a comonad \(H\) on \(\text{Bicat}_s\) such that \(\text{Kl}(H) \cong \text{Bicat}\) by construction;
- Then show that \(H \cong (-)'\) as comonads.
Proposition (Coherence for homomorphisms)

The co-Kleisli category of \((-)\)' : \(\mathbf{Bicat}_s \to \mathbf{Bicat}_s\) is isomorphic to the category \(\mathbf{Bicat}\) of bicategories and homomorphisms.

Proof.

- First define a comonad \(H\) on \(\mathbf{Bicat}_s\) such that \(\text{Kl}(H) \cong \mathbf{Bicat}\) by construction;
- Then show that \(H \cong \text{(-)'}\) as comonads.

Explicitly, given bicategory \(\mathcal{B}\), we form \(H\mathcal{B}\) as follows:

Start with \(FU\mathcal{B}\) as above. Given \(f : X \to Y\) in \(\mathcal{B}\), write \([f] : X \to Y\) for corresponding generator in \(FU\mathcal{B}\).
Now adjoin 2-cells to $FUB$ as follows:

- For each $\alpha : f \Rightarrow g$ in $B$, a 2-cell
  \[ [\alpha] : [f] \Rightarrow [g] ; \]

- For each $X \in B$, a 2-cell
  \[ \eta_X : \text{id}_X \Rightarrow [\text{id}_X] ; \]

- For each $X \xrightarrow{f} Y \xrightarrow{g} Z \in B$, a 2-cell
  \[ \mu_{g,f} : [g] \circ [f] \Rightarrow [g \circ f] ; \]
Now adjoin 2-cells to $FU\mathcal{B}$ as follows:

- For each $\alpha: f \Rightarrow g$ in $\mathcal{B}$, a 2-cell
  \[[\alpha]: [f] \Rightarrow [g];\]

- For each $X \in \mathcal{B}$, a 2-cell
  \[\eta_X: \text{id}_X \Rightarrow [\text{id}_X];\]

- For each $X \xrightarrow{f} Y \xrightarrow{g} Z \in \mathcal{B}$, a 2-cell
  \[\mu_{g,f}: [g] \circ [f] \Rightarrow [g \circ f];\]

And quotient out the 2-cells by equations making:

- $[-]$ be functorial on 2-cells;
- $\mu_{g,f}$ be natural in $g$ and $f$;
- The $\mu_{g,f}$’s and $\eta_X$’s satisfy the unit and associativity laws.

The result of this is $H\mathcal{B}$. 
By construction, maps $H\mathcal{B} \to \mathcal{C}$ are in bijection with homomorphisms $\mathcal{B} \to \mathcal{C}$. 
By construction, maps $H \mathcal{B} \to \mathcal{C}$ are in bijection with homomorphisms $\mathcal{B} \to \mathcal{C}$.

We can now make $H$ into a comonad so that $\textbf{Kl}(H) \cong \textbf{Bicat}$ (comonad structure on $H$ is combinatorial essence of composition of homomorphisms—compare [Hess-Parent-Scott 2006]).
By construction, maps $H \mathcal{B} \to \mathcal{C}$ are in bijection with homomorphisms $\mathcal{B} \to \mathcal{C}$.

We can now make $H$ into a comonad so that $\textbf{Kl}(H) \cong \textbf{Bicat}$ (comonad structure on $H$ is combinatorial essence of composition of homomorphisms—compare [Hess-Parent-Scott 2006]).

Finally, we show that $H \cong (\cdot)'$ as comonads (a normalization proof).
Weak maps of tricategories

Consider the category $\text{Tricat}_s$:

- **Objects** are tricategories;
- **Morphisms** are strict homomorphisms.

We use an algebraic definition of tricategory, so $\text{Tricat}_s$ is l.f.p. and in particular cocomplete.
Weak maps of tricategories

Consider the category $\text{Tricat}_s$:

- Objects are tricategories;
- Morphisms are strict homomorphisms.

We use an algebraic definition of tricategory, so $\text{Tricat}_s$ is l.f.p. and in particular cocomplete.

No-one has written down the cofibrantly generated model structure on $\text{Tricat}_s$ yet, but it should have:

- Weak equivalences being triequivalences;
- Every object being fibrant.

Can we describe the corresponding weak maps?
Yes: because we can describe the cofibrant replacement comonad \((-)\)'.
It’s generated by the following set of maps in $\text{Tricat}_s$: 

\[
\begin{array}{cccc}
\emptyset & \bullet & \bullet & \bullet \\
\bullet & \rightarrow & \bullet & \bullet \\
\bullet & \rightarrow & \bullet & \bullet \\
\end{array}
\]
Explicitly, if $\mathcal{T}$ is a tricategory, then $\mathcal{T}'$ is given as follows:

- Ignore the 2- and 3-cells and form the free tricategory $F_{\mathcal{T}}\mathcal{T}$ on the underlying 1-graph of $\mathcal{T}$. Write $\varepsilon: F_{\mathcal{T}}\mathcal{T} \rightarrow \mathcal{T}$ for the counit map.
Explicitly, if \( \mathcal{T} \) is a tricategory, then \( \mathcal{T}' \) is given as follows:

- Ignore the 2- and 3-cells and form the free tricategory \( FU\mathcal{T} \) on the underlying 1-graph of \( \mathcal{T} \). Write \( \varepsilon: FU\mathcal{T} \to \mathcal{T} \) for the counit map.
- For each pair of 1-cells \( f, g: X \to Y \) in \( FU\mathcal{T} \) and each 2-cell \( \alpha: \varepsilon(f) \Rightarrow \varepsilon(g) \) in \( \mathcal{T} \), adjoin a 2-cell \( (f, g, \alpha): f \Rightarrow g \) to \( FU\mathcal{T} \). Call the result \( \mathcal{T}^{\#} \), and write \( \varepsilon^{\#}: \mathcal{T}^{\#} \to \mathcal{T} \) for the induced counit.
Explicitly, if $\mathcal{T}$ is a tricategory, then $\mathcal{T}'$ is given as follows:

- Ignore the 2- and 3-cells and form the free tricategory $FUT$ on the underlying 1-graph of $\mathcal{T}$. Write $\epsilon: FUT \to \mathcal{T}$ for the counit map.
- For each pair of 1-cells $f, g: X \to Y$ in $FUT$ and each 2-cell $\alpha: \epsilon(f) \Rightarrow \epsilon(g)$ in $\mathcal{T}$, adjoin a 2-cell $(f, g, \alpha): f \Rightarrow g$ to $FUT$. Call the result $\mathcal{T}^\#$, and write $\epsilon^\#: \mathcal{T}^\# \to \mathcal{T}$ for the induced counit.
- Factorise $\epsilon^#$ as

$$
\mathcal{T}^\# \xrightarrow{a} \mathcal{T}' \xrightarrow{b} \mathcal{T}
$$

where $a$ is bijective on 0-, 1- and 2-cells and $b$ is locally fully faithful.
As before \((-)\)' underlies a comonad, so we obtain a category $\mathbf{Kl}_{(-)' \prime}$ of "tricategories and weak maps".
As before \((-\cdot)^\prime\) underlies a comonad, so we obtain a category \(\textbf{Kl}_{(-)^\prime}\) of “tricategories and weak maps”.

A priori quite surprising, because trihomomorphisms à la [Gordon-Power-Street 1995] do not compose associatively: there is no category of tricategories and (ordinary) trihomomorphisms.
As before \((-)\)' underlies a comonad, so we obtain a category \(\mathbf{Kl}_{(-)'}\) of “tricategories and weak maps”.

A priori quite surprising, because trihomomorphisms à la [Gordon-Power-Street 1995] do not compose associatively: there is no category of tricategories and (ordinary) trihomomorphisms.

So what do these new weak morphisms look like? Can they really be as weak as trihomomorphisms?
Definition

An unbiased trihomomorphism $F: \mathcal{T} \to \mathcal{U}$ is given by:

- For each object $X \in \mathcal{T}$, an object $FX \in \mathcal{U}$;
- For each 1-cell $f: X \to Y \in \mathcal{T}$, a 1-cell $Ff: FX \to FY$ in $\mathcal{U}$;

Plus...
For every bracketed pasting diagram

\[ A := W \]

in \( \mathcal{T} \), a 2-cell

\[ \text{in } \mathcal{U}. \]
For every pair of bracketed pasting diagrams \( A, B \) with the same boundary in \( \mathcal{T} \); i.e.,

\[
\begin{array}{c}
X_1 \xrightarrow{f_1} \cdots \xrightarrow{f_{m-1}} X_m \\
\downarrow \alpha & \downarrow \beta \\
Y_1 \xrightarrow{g_1} \cdots \xrightarrow{g_{n-1}} Y_n
\end{array}
\]

and for every 3-cell \( \Gamma : \alpha \Rightarrow \beta \) between them, a 3-cell

\[ F\Gamma : F(A) \Rightarrow F(B) \]

in \( \mathcal{U} \).
For every pair of composable bracketed pasting diagrams $A$, $B$ in $\mathcal{T}$; i.e.,

\[
\begin{array}{c}
W_1 \xrightarrow{f_1} \cdots \xrightarrow{f_{m-1}} W_m \\
V \xrightarrow{g_0} X_1 \xrightarrow{g_1} \cdots \xrightarrow{g_{n-1}} Y_n \xrightarrow{g_n} Z
\end{array}
\]

\[
\begin{array}{c}
\Downarrow \alpha \\
\Downarrow \beta \\
Y_1 \xrightarrow{h_0} \cdots \xrightarrow{h_{k-1}} Y_k \xrightarrow{h_k} \end{array}
\]

a 3-cell

$$\mu_{A,B} : F(B) \cdot F(A) \Rightarrow F(B \cdot A)$$

in $\mathcal{U}$. 
For every identity pasting diagram $\text{id}_{\{f_i\}}$, i.e.

$$
\begin{array}{c}
Y_1 \xrightarrow{f_1} \cdots \xrightarrow{f_{m-1}} Y_m \\
\downarrow \text{id} \\
X \xrightarrow{f_0} Y_1 \xleftarrow{f_1} \cdots \xleftarrow{f_{m-1}} Y_m
\end{array}
$$

in $\mathcal{T}$, a 3-cell

$$
\eta_{\{f_i\}} : \text{id}_{\{Ff_i\}} \Rightarrow F(\text{id}_{\{f_i\}})
$$

in $\mathcal{U}$. 
For every pair $A, B$ of horizontally composable bracketed pasting diagrams:

\[
\begin{array}{c}
U_1 \xrightarrow{f_1} \cdots \xrightarrow{f_{m-1}} U_m \\
\downarrow \alpha \\
V_1 \xrightarrow{g_1} \cdots \xrightarrow{g_{n-1}} V_n \\
\downarrow k_1 \\
T \\
\end{array}
\quad
\begin{array}{c}
X_1 \xrightarrow{h_1} \cdots \xrightarrow{h_{k-1}} X_k \\
\downarrow \beta \\
Y_1 \xrightarrow{k_1} \cdots \xrightarrow{k_{l-1}} Y_l \\
\downarrow \gamma \\
W \\
\end{array}
\]

in $\mathcal{T}$, a 3-cell

\[
\gamma_{A,B} : F(B) \otimes F(A) \Rightarrow F(B \otimes A)
\]

in $\mathcal{U}$. 
All such that ten straightforward axioms hold.
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**Proposition (Coherence for unbiased trihomomorphisms)**

The co-Kleisli category of \((-)' : \text{Tricat}_s \to \text{Tricat}_s\) is isomorphic to the category \(\text{UTricat}\) of tricategories and unbiased trihomomorphisms.
All such that ten straightforward axioms hold.

**Proposition (Coherence for unbiased trihomomorphisms)**

The co-Kleisli category of \((-)') : \textbf{Tricat}_s \rightarrow \textbf{Tricat}_s is isomorphic to the category \textbf{UTricat} of tricategories and unbiased trihomomorphisms.

**Proof.**

- First define a comonad \(H\) on \(\textbf{Tricat}_s\) such that \(\text{Kl}(H) \cong \textbf{UTricat}\) by construction;
- Then show that \(H \cong (-)'\) as comonads.
But are unbiased trihomomorphisms as weak as ordinary ones?
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**Proposition**

*Every unbiased trihomomorphism $\mathcal{T} \rightarrow \mathcal{U}$ gives rise to an ordinary trihomomorphism; and vice versa.*
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**Proposition**

*Every unbiased trihomomorphism* $\mathcal{T} \rightarrow \mathcal{U}$ *gives rise to an ordinary trihomomorphism; and vice versa.*

E.g., given an unbiased trihomomorphism $F: \mathcal{T} \rightarrow \mathcal{U}$ let us show that it preserves 1-cell composition up to equivalence.
Let $X \xrightarrow{f} Y \xrightarrow{g} Z$ in $\mathcal{T}$. We have bracketed pasting diagrams $A, A^* :=$

in $\mathcal{T}$, and so obtain 2-cells

in $\mathcal{U}$. Moreover, $A \circ A^* = \text{id}$ implies $F(A) \circ F(A)^* \cong \text{id}$ and dually.
Can formalise the above equivalence using a *bicategory of tricategories*.
Can formalise the above equivalence using a *bicategory of tricategories*.

**Definition (G.-Gurski 2008)**

Given ordinary trihomomorphisms $F, G : \mathcal{T} \to \mathcal{U}$, a *tricategorical icon* $\Gamma : F \Rightarrow G$:

- Exists only if $F$ and $G$ agree on 0- and 1-cells;
- Is then given by 3-cells $\Gamma_\alpha : F\alpha \Rightarrow G\alpha$ for each 2-cell $\alpha \in \mathcal{T}$;
- Plus some coherence data.
Can formalise the above equivalence using a *bicategory of tricategories*.

**Definition (G.-Gurski 2008)**

Given ordinary trihomomorphisms $F, G : \mathcal{T} \to \mathcal{U}$, a *tricategorical icon* $\Gamma : F \Rightarrow G$:

- Exists only if $F$ and $G$ agree on 0- and 1-cells;
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- Plus some coherence data.

Similar definition of *unbiased tricategorical icon*. 
Proposition

There is a bicategory $\text{Tricat}_2$ with:

- Objects being tricategories;
- Morphisms being (ordinary) trihomomorphisms;
- 2-cells being tricategorical icons.
Proposition

There is a bicategory $\text{Tricat}_2$ with:

- Objects being tricategories;
- Morphisms being (ordinary) trihomomorphisms;
- 2-cells being tricategorical icons.

There is also a 2-category $\text{UTricat}_2$ with:

- Objects being tricategories;
- Morphisms being unbiased trihomomorphisms;
- 2-cells being unbiased tricategorical icons.
Proposition

The bicategory \( \text{Tricat}_2 \) is equivalent to the 2-category \( U\text{Tricat}_2 \).
Proposition

The bicategory $\text{Tricat}_2$ is equivalent to the 2-category $\text{UTricat}_2$.

Proof.

- First extend the category $\text{Tricat}_s$ to a 2-category, with icons as 2-cells;
- Then define a 2-comonad $H$ on $\text{Tricat}_s$ such that $Kl(H) \cong \text{UTricat}_2$ by construction;
- Then define a pseudo-comonad $K$ on $\text{Tricat}_s$ such that $Kl(K) \cong \text{Tricat}_2$ by construction;
- Finally show that $H \simeq K$ as pseudo-comonads.
Weak maps of weak $\omega$-categories

Consider the category $\omega$-$\text{Cat}_s$:

- *Objects* are (algebraic) weak $\omega$-categories;
- *Morphisms* are strict homomorphisms.

We can play the same game as before to obtain a category $\omega$-$\text{Cat}$ of weak $\omega$-categories and weak homomorphisms.
This time the cofibrant replacement comonad \((-)'\) is generated by the following set of maps in \(\omega\text{-Cat}_s\):

\[
\begin{array}{cccccc}
\emptyset & \bullet & \bullet & \bullet & \bullet & \bullet \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\bullet & \rightarrow & \bullet & \rightarrow & \bullet & \rightarrow \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\end{array}
\]
Explicitly, \((-\)' : \(\omega\text{-Cat}_s \to \omega\text{-Cat}_s\) is the comonad arising from the adjunction

\[
\omega\text{-Cptd} \xleftrightarrow{U \atop \top} \omega\text{-Cat}_s
\]

where \(\omega\text{-Ctpd}_s\) is the category of \(\omega\)-computads.

And by now natural to define...
Explicitly, \((-\)'): \(\omega\text{-Cat}_s \rightarrow \omega\text{-Cat}_s\) is the comonad arising from the adjunction

\[
\begin{array}{c}
\omega\text{-Cptd} \\
\downarrow^U \downarrow \rightarrow \end{array} \quad \begin{array}{c}
\omega\text{-Cat}_s \\
\uparrow^F \uparrow \leftarrow \end{array}
\]

where \(\omega\text{-Cptd}_s\) is the category of \(\omega\)-computads.

And by now natural to define...

**Definition**

The category \(\omega\text{-Cat}\) of weak \(\omega\)-categories and weak morphisms is the co-Kleisli category of \((-\)'): \(\omega\text{-Cat}_s \rightarrow \omega\text{-Cat}_s\)
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