1. Find the regions in the $xy$ plane where the equation
\[(1 + x) u_{xx} + 2xy u_{xy} - y^2 u_{yy} = 0\]
is elliptic, hyperbolic or parabolic; sketch the regions.

2. Find the general solution $u = u(x, y)$ of the following equations:
   (i) $u_{xx} - 4u_{xy} + 3u_{yy} = 0$, (ii) $u_{xx} - 4u_{xy} + 4u_{yy} = 0$.

3. Consider the second order linear PDE
   
   \[a_{11}u_{xx} + 2a_{12}u_{xy} + a_{22}u_{yy} + a_1u_x + a_2u_y + a_0u = 0,\]
   where $a_{11}, a_{12}, a_{22}, a_1, a_2$ and $a_0$ are constants. In lectures it was shown that when this
   PDE is elliptic, a suitable linear transformation of the independent variables transforms
   the PDE to the form
   
   \[u_{xx} + u_{yy} + \text{lower order terms} = 0.\]
   Show that when the PDE is hyperbolic or parabolic, suitable linear transformations of
   the independent variables transform the PDE to the forms
   \[u_{xx} - u_{yy} + \text{lower order terms} = 0\]
   or
   \[u_{xx} + \text{lower order terms} = 0,\]
   respectively.

4. Use the method of separation of variables to find the solution $u = u(x, t)$ to the wave
   equation
   
   \[u_{tt} = c^2 u_{xx}, \quad 0 < x < l, \ t \geq 0,\]
   subject to the boundary conditions
   
   \[u(0, t) = u(l, t) = 0,\]
   and the initial conditions
   \[u(x, 0) = \sin \frac{\pi x}{l} + \frac{1}{3} \sin \frac{3\pi x}{l}, \quad u_t(x, 0) = 0.\]
   ($c$ and $l$ are fixed positive constants.)

5. Use the method of separation of variables to find the solution $u = u(x, t)$ to the heat
   equation
   
   \[u_t = ku_{xx}, \quad 0 < x < l, \ t \geq 0,\]
   subject to the boundary conditions
   
   \[u(0, t) = u(l, t) = 0,\]
   and the initial condition
   \[u(x, 0) = x (l^2 - x^2).\]
   ($k$ and $l$ are fixed positive constants.)