(1) For each of the following functions \( f \), first write down the derivative of \( f \) at \( x_0 \), \( f'(x_0) \), then calculate the slope of the tangent to the graph of \( f \) at \( (1, f(1)) \) and check it by sketching the graph of \( f \) and drawing the tangent line at \( (1, f(1)) \).

(a) \( f(x) = x^4 \)  
(b) \( f(x) = 2x^{1/2} \)  
(c) \( f(x) = (x-1)^2 \).

(2) Find where the tangent lines to the graph of \( y = x^2 \) through \((-1,1)\) and \((3,9)\) intersect. Set this up as simultaneous linear equations and use matrix inversion to solve it.

(3) The **normal line** to a curve \( C \) at a point \( P \) is defined to be that line which passes through \( P \) and is perpendicular to the tangent line to \( C \) at \( P \). Where else does the normal line to the parabola \( y = x - x^2 \) at the point \((1,0)\) intersect the parabola?

(4) Suppose that \( \tan \alpha = -\sqrt{3} \) and \( \alpha \) is in the second (top left) quadrant, find \( \cos \alpha \).

(5) When viewed from a distance, the angle of elevation to the top of a radio antenna is \( \pi/6 \) radians. After the observer moves 100 metres closer to the antenna the angle of elevation is \( \pi/4 \) radians. Let \( h \) be the height of the antenna (in metres) and \( x \) the distance of the first observation from the base of the antenna. Then \( \tan(\pi/6) = h/x \) and \( \tan(\pi/4) = h/(x-100) \). Find the height of the antenna.

(6) For each of the following equations, find all the solutions \( \theta \) with \( 0 \leq \theta < 2\pi \).

(a) \( \tan \theta = -1 \),  
(b) \( \cos \theta = -\sqrt{3}/2 \).

(7) Verify each of the following identities:

(a) \( \sin x \cot x \sec x = 1 \),  
(b) \( \frac{16 \sin^2 x - 1}{4 \sin x + 1} = 4 \sin x - 1 \),  
(c) \( \frac{\sin^2 x - 3 \sin x + 2}{\sin x - 1} = \sin x - 2 \),