1. Determine bases for the following subspaces of $\mathbb{R}^3$.
   (a) The plane $3x - 2y + 5z = 0$.
   (b) The line $x = 2t, y = -t, z = 4t$.

2. Show that the set $W$ of polynomials in $P_2$ such that $p(1) = 0$ is a subspace of $P_2$.
   (a) Make a conjecture about the dimension of $W$. Can you explain yourself in any way?
   (b) Confirm (or reject) your conjecture by finding a basis for $W$.

3. Prove that the row vectors of an $n \times n$ invertible matrix $A$ form a basis for $\mathbb{R}^n$.

4. Here are some definitions.
   The row space of a matrix $A$ is the space spanned by its rows; the dimension of this space is called $\text{rank}(A)$.
   The null space of a matrix $A$ is the solution set of the homogeneous system $A\mathbf{x} = \mathbf{0}$; the dimension of this space is called $\text{nullity}(A)$.

   Let
   $$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

   Find a basis for the row space and a basis for the null space. Verify the equation
   \[
   \text{rank}(A) + \text{nullity}(A) = n
   \]
   where $n$ is the number of columns of $A$. Can you try and explain why this equation will be true for any $A$?

5. Given that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are are linearly independent set of vectors in $\mathbb{R}^3$, explain whether or not
   $$\{\vec{v}_1 + \vec{v}_2 + \vec{v}_3, \vec{v}_1 + 2\vec{v}_2 + 3\vec{v}_3, \vec{v}_1 - \vec{v}_2 - 2\vec{v}_3\}$$
   form a linearly independent set.

6. Let $(a, b)$ and $(c, d)$ be two elements of $\mathbb{R}^2$. Show that if $ad - bc = 0$ then they are linearly dependent, and that if $ad - bc \neq 0$ then they are linearly independent.
7. Does \( \sum_{n=2}^{\infty} \frac{1}{\log n} \) converge or diverge?

8. By comparing to an improper integral, decide whether the series

\[
\frac{1}{2} + \frac{2}{5} + \frac{2}{8} + \cdots + \frac{n}{n^2 + 1} + \cdots
\]

converges or diverges.

9. Show that \( \sum \frac{1}{n!} \) converges (using the ratio test). Show that \( \sum \frac{n^p}{n!} \) converges, where \( p \) is an arbitrary constant.

10. Give an example of a diverging alternating series.

11. Explain why, in a converging alternating series, the approximation to the limiting sum by summing up to a certain point has an error less than the absolute value of the next term. If the sum \( \sum (-1)^{n+1} \frac{1}{2np!} \) is approximated by summing the first 6 terms, what is the most this approximation can differ by from the exact limiting sum? Is the approximation an under or over estimate?

12. Find someone in the class who thinks that \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) converges and explain to them why it doesn’t.

13. For which positive integers \( k \) is the series \( \sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!} \) convergent?