1. Use the comparison theorem to show \( \int_7^\infty \frac{\log 2}{x^2 + 2x} \, dx < \frac{1}{128} \).

2. The rate, \( r \), at which people get sick during an epidemic of the flu can be approximated by \( r = 1000e^{-0.5t} \), where \( r \) is measured in people per day and \( t \) is measured in days since the start of the epidemic.
   
   (a) Sketch a graph of \( r \) as a function of \( t \). Use \( r' \) to tell you where the graph is increasing/decreasing, and use \( r'' \) to tell you where the graph is concave up/down.
   
   (b) When are people getting sick fastest?
   
   (c) How many people get sick altogether?

3. Is the area under the curve \( y = \frac{1}{\cos^2 x} \) well defined between \( x = 0 \) and \( x = \pi/2 \)?

4. The Gamma function, \( \Gamma \), is defined by \( \Gamma(x) = \int_0^\infty t^{x-1}e^{-t} \, dt \). Evaluate \( \Gamma(1) \) and \( \Gamma(2) \). Integrate by parts to show for \( n \in \mathbb{N} \), we have \( \Gamma(n + 1) = n\Gamma(n) \). Find a nice formula for \( \Gamma(n) \).

5. Evaluate the sum
   \[
   S = 1 + (2z) + (2z)^2 + (2z)^3 + \cdots + (2z)^N
   \]
   Under what circumstances is \( S \) well defined in the limit \( N \to \infty \)?

6. The harmonic series is
   \[
   \sum_{n=1}^\infty \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots
   \]
   The partial sums of the harmonic series are \( s_n = \sum_{i=1}^n \frac{1}{i} \). Show that \( s_{2k} > 1 + \frac{k}{2} \). Hint: \( \frac{1}{3} + \frac{1}{4} > \frac{1}{4} + \frac{1}{4} \), and \( \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} > \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \).
   Does the harmonic series converge?

7. The harmonic series diverges, though extremely slowly. Using a right hand sum (in the integral test) we can show that
   \[
   \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} < \log n
   \]
   If a computer can add a million terms of the harmonic series each second, estimate the sum after 1 year, and 2 years.

8. Show that the set of complex numbers which are roots of the equation \( f(z) = z^6 - 1 \) forms a group under multiplication. What is the order of this group? Hint: use the polar form of complex numbers.

9. Construct the group table for \( \mathbb{Z}_6 \) under addition. Check that it is a group.
10. Let \( D_6 = \{ e, \rho, \rho^2, \phi, \rho\phi, \rho^2\phi \} \), the group of symmetries of an equilateral triangle. In this representation, each of the elements of \( D_6 \) is a \( 2 \times 2 \) matrix, the binary operation is matrix multiplication, and \( e \) is the identity matrix. (In addition, you may find it useful that \( \rho \) represents an anticlockwise rotation of \( \frac{2\pi}{3} \) and \( \phi \) represents a reflection in the vertical axis.) The group table for \( D_6 \) is shown below.

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</table>

(a) How can you tell by inspecting the group table for \( D_6 \) that each element has an inverse?

(b) Is \( D_6 \) Abelian? Provide reasons to justify your answer.

(c) Determine the order of each of the elements of \( D_6 \).

(d) Is \( D_6 \) cyclic?

(e) List all six subgroups of \( D_6 \).

(f) Explain why it is impossible for \( (\mathbb{Z}_6, +) \) to be isomorphic with \( D_6 \).

11. What is the difference between the order of an element of a group and the order of a group?

12. Give an example of a group of order 4 which is not cyclic. Explicitly show that your example is not cyclic by considering the orders of each of the elements.

13. Show that a group \( G \) of order 3, say with elements \( \{ a, b, c \} \) must be Abelian. If two groups are said to be distinct if they are not isomorphic, how many distinct groups of order 3 are there?