1. Draw a contour map for each function, showing several level curves.
   (a) \( f(x, y) = x^2 + y^2 \)
   (b) \( f(x, y) = \frac{x}{y} \)
   (c) \( f(x, y) = \sqrt{x + y} \)

2. HH, page 582 q10 and page 583 q17 (Contour diagrams)

3. Compare the contour maps of the functions \( f(x, y) = x^2 + 9y^2 \) and \( g(x, y) = \sqrt{36 - 9x^2 - 4y^2} \). As an additional exercise, sketch the domain of \( g \).

4. HH page 601 q22 & 23

5. Two economics researchers, Charles Cobb and Paul Douglas, developed a model of the production output \( P \) for the American economy as a function of the labour \( L \) and the amount of capital invested \( K \). The function they used is
   \[
   P = f(L, K) = bL^\alpha K^{1-\alpha}
   \]
   where \( P \) is the monetary value of all goods produced in one year, \( L \) is the amount of labour measured in total person hours worked in one year, and \( K \) is the capital invested in dollars and \( b \) is a constant.
   
   Show that for \( b = 1.01 \) and \( \alpha = 0.75 \) then the production doubles if both the amount of labour and the capital investment doubles. Is this also true for general \( b \) and \( \alpha \)?

6. Evaluate the limit or explain why it does not exist.
   (a) \[
   \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}
   \]
   (b) \[
   \lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2 + y^2}
   \]
   (c) \[
   \lim_{(x,y)\to(0,0)} \frac{x^3 + xy^2}{x^2 + y^2}
   \]
   (d) \[
   \lim_{(x,y)\to(0,0)} \frac{xy + 1}{x^2 + y^2 + 1}
   \]

7. The sales of a product, \( S = f(p, a) \) is a function of the price, \( p \) of the product (in dollars per item) and the amount of advertising, \( a \), in thousands of dollars.
   (a) Do you expect \( f_p \) to be positive or negative? Why?
   (b) Explain the meaning of \( f_a(8, 12) = 150 \) in terms of sales.

8. HH page 645 q13

9. HH page 646 Q17

10. Let \( f(x, y) = \frac{x + y}{1 + x^2} \). Find the equation of the tangent plane and the normal line at \((1, -2)\). Find the directional derivative at \((1, -2)\) in the direction of \((-1, 4)\).
11. Prove \[ \frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1} \]
by using the principle of mathematical induction. Also try to prove it directly by summing the series, using the fact \[ \frac{1}{3\cdot4} = \frac{1}{3} - \frac{1}{4}. \]

12. Prove \[ \sum_{k=1}^{n} \frac{1}{\sqrt{k}} \geq \sqrt{n} \] by using the principle of mathematical induction. (Hence the sum \[ \frac{1}{1} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \] diverges as \( n \to \infty \).)

13. Use the principle of mathematical induction to show that \[ \sum_{i=1}^{n} i^3 = \frac{1}{4}n^2(n+1)^2 \]

14. Let \( a_n = \sum_{k=0}^{n-1} \frac{1}{k!} \) and \( b_n = 1 + \sum_{k=0}^{n} \frac{1}{2^k} \).
   (a) Explain why \( (a_n) \) is an increasing sequence.
   (b) Explain why \( a_n < b_n \) for \( n \in \mathbb{N} \).
   (c) Find a closed expression for \( b_n \). Hint: think geometric series.
   (d) Explain why \( (a_n) \) is bounded and give a brief justification of why it converges. What does \( (a_n) \) converge to? (You will need to recognize this famous number rather than work it out.)

15. Find a real sequence \( x_n \) that satisfies the following conditions simultaneously:
   (a) \( 0 < x_n < 1 \) for every \( n \in \mathbb{N} \);
   (b) \( x_n \neq \frac{1}{2} \) for every \( n \in \mathbb{N} \) and;
   (c) \( x_n \to \frac{1}{3} \) as \( n \to \infty \).

16. You will now prove that if a sequence \( (a_n) \) converges to a limit, then the limit is unique. Assume that \( s_n \to a \) as \( n \to \infty \) and also \( s_n \to b \) as \( n \to \infty \). First of all, say what these two convergence statements say in terms of the \( \varepsilon-N \) definition. Then consider \(|a-b|\) and use the identity \( a-b = a - s_n + s_n - b \).

17. Let \( a_1 = 2 \) and \( a_{n+1} = \frac{1}{3-a_n} \).
   (a) Work out the first few terms of \( (a_n) \).
   (b) Show that \( 0 < a_n \leq 2 \). Use the principle of mathematical induction (or otherwise). Can you do any better and show \( \frac{1}{3} < a_n \leq 2? \)
   (c) Show that \( (a_n) \) is decreasing. Use the principle of mathematical induction and consider \( \frac{a_{n+1}}{a_n} \).
   (d) Explain why \( (a_n) \) converges to a limit. Can you make a conjecture for the limit?
   (e) Use the arithmetic of limits to evaluate the limit of \( (a_n) \).

18. Find the limit of the sequences or explain why the limit doesn’t exist. Use whatever methods you feel like but provide justification.
   (a) \( 2 + \sin \frac{n\pi}{2} \)
   (b) \( 2 + \frac{\cos n\pi}{n} \)
   (c) \( \frac{\sqrt{n}}{1+\sqrt{n}} \)
   (d) \( \frac{1+2+3+\ldots+n}{n^2} \)