1. Discuss the reduced row echelon form of

\[
\begin{pmatrix}
a & b & c \\
d & e & f \\
g & h & i \\
\end{pmatrix}
\]

Indicate whether the statement is always true or sometimes false. Justify your answer by giving a logical argument or a counterexample. It is not enough to only provide an example for a statement which you claim to be true.

(a) A linear system of three equations in five unknowns must be consistent.

(b) A linear system of three equations in five unknowns cannot be consistent.

(c) If a linear system of \( n \) equations in \( n \) unknowns has \( n \) leading 1’s in the reduced row echelon form of its augmented matrix, then the system has exactly one solution.

(d) If a linear system of \( n \) equations in \( n \) unknowns has two equations that are multiples of one another, then the system is inconsistent.

2. Consider the matrices

\[
A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\]

(a) Show that the equation \( A\mathbf{x} = \mathbf{x} \) can be rewritten \((A - I)\mathbf{x} = 0\) and use this result to solve \( A\mathbf{x} = \mathbf{x} \) for \( \mathbf{x} \).

(b) Solve \( A\mathbf{x} = 4\mathbf{x} \).

3. Determine whether or not \((8, -6, 4)\) is a linear combination of \((3, 1, 0), (-1, 2, 1), \text{and} (1, -1, 2)\). Can \((c_1, c_2, c_3)\) always be written as a linear combination of the three vectors given for arbitrary real numbers \(c_1, c_2\) and \(c_3\)?

4. Let \( A\mathbf{x} = \mathbf{b} \) be a consistent system of linear equations, and let \( \mathbf{x}_1 \) be a solution. Show that every solution to the system can be written in the form \( \mathbf{x} = \mathbf{x}_1 + \mathbf{x}_0 \) where \( \mathbf{x}_0 \) is a solution to \( A\mathbf{x} = 0 \). Also, show that every vector of the form \( \mathbf{x} = \mathbf{x}_1 + \mathbf{x}_0 \) is a solution.

5. Shortest ever review of differentiation.

(a) What are the derivatives of the fundamental functions \(x^r, e^x, a^x, \log x\) (this is base e), \(\sin x, \cos x\)?

(b) Write down a formula that says “the derivative of a sum of functions is the sum of the derivatives.”

(c) Write down a formula that says “the derivative of a constant times a function is the constant times the derivative.”

(d) What is the product rule? Give an example.

(e) What is the quotient rule? Give an example.

(f) What is the chain rule? Give six examples of functions which can be written as compositions, say what the compositions are, then differentiate.

6. As a warm up exercise, and to make sure you haven’t forgotten everything, here is a graph sketching exercise. We will be using the facts for a function \(f : \mathbb{R} \rightarrow \mathbb{R}\)

- \(f\) is increasing when \(f' > 0\)
- \(f\) is decreasing when \(f' < 0\)
\[ f(x) = e^{-x^2} \]. Calculate \( f' \) and \( f'' \) and use this information to sketch a careful graph of \( f \). Locate the coordinates of the inflection points.

7. A straight line is determined by a point and the slope. Explain why, doing the minimum amount of work, that the equation of the tangent to a function \( f \) at the point \((a, f(a))\) is given by \( y = f'(a)(x - a) + f(a) \). Drawing a graph of a possible function and the tangent at some point should help. (This form for the formula for the tangent line will be useful later, in functions of two variables.)

8. Determine whether each statement is true or false.
   (a) Two lines parallel to a third line are parallel.
   (b) Two lines perpendicular to a third line are parallel.
   (c) Two planes parallel to a third plane are parallel.
   (d) Two planes perpendicular to a third plane are parallel.
   (e) Two lines parallel to a plane are parallel.
   (f) Two lines perpendicular to a plane are parallel.
   (g) Two planes parallel to a line are parallel.
   (h) Two planes perpendicular to a line are parallel.
   (i) Two lines either intersect or are parallel.
   (j) Two planes either intersect or are parallel.
   (k) A plane and a line either intersect or are parallel.

9. Find a vector equation and parametric equations for the line.
   (a) The line through the point \((1, 0, -3)\) and parallel to the vector \((2, -4, 5)\).
   (b) The line through the origin and parallel to the line \(x = 2t, y = 1 - t\) and \(z = 4 + 3t\).

10. Find an equation of the plane
    (a) The plane through the point \((4, 0, -3)\) with normal vector \((0, 1, 2)\).
    (b) The plane through the point \((-2, 8, 10)\) and perpendicular to the line \(x = 1 + t, y = 2t\) and \(z = 4 - 3t\).

11. If \(a, b,\) and \(c\) are all not zero show that the equation \(ax + by + cz + d = 0\) represents a plane and \((a, b, c)\) is a vector normal to the plane. Hint: suppose that \(a \neq 0\) and rewrite the equation in the form
    \[ a \left( x + \frac{d}{a} \right) + b(y - 0) + c(z - 0) = 0 \]

12. The triangle inequality for vectors is
    \[ |\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}| \]
    (a) Give a geometric interpretation of the Triangle Inequality.
    (b) Use the Cauchy-Schwarz Inequality and the definition of the dot product to prove the Triangle Inequality.

**Cauchy-Schwarz Inequality:** \[ |\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}| |\mathbf{b}| \]

**Definition of the dot product:** \( \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta \) where \( \theta \) is the angle between \( \mathbf{a} \) and \( \mathbf{b} \).