1. A detective finds a murder victim at 9am. (This problem is fictional.) The temperature of the body is 90.3°F. One hour later, the temperature of the body is 89.0°F. The temperature of the room is constant at 68°F.

   (a) Assuming the temperature, $T$, of the body obeys Newton’s Law of Cooling, write a differential equation for $T$. Newton’s Law of Cooling states that the rate of change of the temperature is proportional to the temperature difference between the temperature of the object and the temperature of the surroundings.

   (b) Solve the differential equation to estimate the time the murder occurred. Hint: you need to know something about bodies.

2. Find the general solution of

   (a) \[ xy \frac{dy}{dx} = \frac{x^2 + 1}{y^2 - 1} \]

   (b) \[ (x + 1) \frac{dy}{dx} - 3y = (x + 1)^5 \]

3. (a) Find the general solution of

   i. \[ \frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0. \]

   ii. \[ \frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^{3y}. \]

   (b) Solve the differential equation \[ \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = \sin x \] where $y(0) = 1$ and $y'(0) = 3$.

   (c) Find a basis for the set of solutions to the equation \[ \frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 0. \]

4. Show that the following differential equation

   \[ y \cos(xy) + x \cos(xy) y' = 0 \]

   is exact and find an equation which implicitly determines the solutions. Use the initial condition $y(1) = \pi/2$ to determine the value of the constant in your formula. Solve explicitly for $y$ as a function of $x$ if possible.

5. Compile a dictionary of words used in the vector space section of this course.

6. Let \( p(x) = x^4 + x^3 + 10x^2 + 9x + 9 \). Evaluate \( p(-3) \) and \( p(3i) \). Find all the solutions to \( p(x) = 0 \).

7. Find the first four terms of \( (1+x)^{1/5} \) and hence evaluate \( \sqrt[5]{1.1} \) with an error of less than 0.000004.