1. Let 
\[ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \quad \bar{x} = \begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix} \]

Express the product $A\bar{x}$ as a linear combination of the columns of $A$.

2. An elementary matrix $E$ is one that can be obtained from $I$ by a single row operation. Let the matrix $B$ be the one obtained from matrix $A \in \mathbb{R}^{m \times n}$ by the same row operation. Then $B = EA$. What size must $I$ be? Verify that this equation holds for all three different types of row operations on the matrix
\[ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \]

Make up your own row operations.

3. Explain why elementary row operations do not change the space spanned by the rows of a matrix. (At the very least, verify that this is true for a matrix of your choice; in checking the statement you should realize why it will be true for any matrix.)

4. Here are some definitions for the following question.

The **row space** of a matrix $A$ is the space spanned by its rows; the dimension of this space is called rank($A$).

The **null space** of a matrix $A$ is the solution set of the homogeneous system $A\bar{x} = \bar{0}$; the dimension of this space is called nullity($A$).

5. Let 
\[ A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix} \]

Find a basis for the row space and a basis for the null space. Verify the equation
\[ \text{rank}(A) + \text{nullity}(A) = n \]

where $n$ is the number of columns of $A$. Can you try and explain why this equation will be true for any $A$?