BOOK REVIEW

History in Mathematics Education – The ICMI Study
Edited by John Fauvel and Jan van Maanen, Kluwer Academic Publishers, Dordrecht, ISBN 0-7923-6399-X, xvii + 437 pages

History in Mathematics Education is a kind of opposite to the 'modern mathematics' of the 1960s. 'Modern mathematics' was constructed from the 'elementary' notions of set and function. History, on the other hand, provides an unbounded collection of sources and starting points from which mathematics emerges, and also reveals lines of heuristic development.

This book is the product of a conference held in April 1998, chaired by John Fauvel and Jan van Maanen, and was written partly at the conference and partly during the succeeding six months. Sixty-two contributors are listed from twenty-six countries. Most of the contributors were involved in Tertiary Education, many of these in teacher education. Only eleven claimed experience of teaching in a secondary school and none claimed experience of teaching in a primary school.

That the heart of these proposals should stem from teacher education is both natural and appropriate. Tertiary Education is generally freer from political control than either Primary or Secondary Education, so the possibility of isolated changes initiated by individuals is greater at that level. Moreover, if a reform of teacher awareness is to be initiated, it is from pre-service training that a programme may most plausibly snowball. But in terms of the content of this book, and in relation to what activities may be appropriate at what level of education there is a blurring of the edges between primary, secondary, upper secondary and tertiary education. Perhaps this is inevitable. The teacher-trainer is concerned with what their pre-service students do today, which will best enhance their awareness for school-teaching tomorrow, so a transfer from tertiary to secondary is assumed. But there are distinctions which can be usefully drawn. For example a specific course in the History of Mathematics as an option alongside other mathematics courses, may most easily be mounted in Tertiary Education.

History is not monochrome, and those who have had their awareness enlarged by history do not all tell the same story. The suggestion that there may be historical facts in mathematics (analogous to the years 1066 for

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Britain or 1492 for America) which ought to be known, is hardly whispered in this book.

One proposal, made especially in chapter 9, is that students read primary sources, in translation if necessary; a proposal perhaps particularly suited to pre-service teacher education. The possible pay-offs are multiple.

(i) Mathematics is seen to be man-made. As an example, Fauvel and van Maanen quote the meeting between Napier and Briggs in their introduction.
(ii) To read and struggle to understand the mathematics of someone with another mind-set, is of immediate relevance to teacher-training.
(iii) When problems from the past are accessible to pupils of today, they may change the awareness of today's pupils of mathematics.

But the recommendation is made with caution. The selection of primary sources is of critical importance. Badly chosen historical material may be as inaccessible as the most abstract mathematics.

Another proposal, made in chapter 3, is that, possibly without mentioning history at all, a mathematics course can be structured with historical development as its guiding light. The point of this is to use a course structure that is more congenial for pupils than a formal one. As examples of such courses, O. Toeplitz' *The Calculus a genetic approach* (1963) and H.M. Edwards' *Fermat's Last Theorem* (1977) are cited. If this seems too global an aim, and to demand too much historical knowledge even for a teacher who is a historical enthusiast, the same idea can be applied on a microscopic level, to address specific known student difficulties. This narrower approach is illustrated in chapter 5. But again, the authors are cautious: "The knowledge of history of mathematics is not sufficient to develop teaching strategies... Such work needs competence both in history and in mathematics education research..." [p. 153]

A third proposal, discussed in sections 2.3 and 8.3, is the comparison of similar aspects of mathematics as seen from different cultures. Number systems and the Pythagorean theorem are prime examples. Number systems are an appropriate study in primary school. Varied proofs of the Pythagorean theorem have been commended by educators without reference to history, so the proposals in this case have excellent provenance.

Historical work may require maths students to use the library and to communicate in ways to which they are not accustomed in maths lessons. Of course a library with good historical resources is bound to be helpful, but the world-wide web is now full of historical resources. This book lists and annotates a wide range of relevant web sites in chapter 10.

The one theoretical perspective which emerges in *History in Mathematics Education* is the notion that *ontogeny recapitulates phylogeny.* This was originally a zoological theory propounded by Ernst Haeckel in the late nineteenth century. *Ontogeny* is the growth of the individual. *Phylogeny* is the development of the species. The theory which Haeckel propounded referred to the comparison between the growth of the embryo (in fish, reptiles and birds, before hatching, and in mammals, before parturition) and the evolution of the species. Various authors, about 1900, suggested that the theory might apply to the development of mathematical knowledge. The timing of this suggestion coincided with the greatest triumphs of axiomatisation and therefore pointed, at an optimal moment, to a development of mathematics which was different from a formal development. The theory is no longer affirmed by zoologists, and the moment its mathematical counterpart is examined in detail, it too breaks down. For example, negative numbers, which today form part of the primary school pupil's diet, were resisted by European mathematicians into the nineteenth century, but were manipulated confidently by Chinese mathematicians two thousand years ago. So while the ontogeny of negative numbers is clear for today's pupils, their phylogeny is irregular. The most careful study of this theory of mathematical knowledge is that by J.Piaget and R.Garcia *Psychogenesis and the History of Science* (1989). Piaget and Garcia's account of the phylogeny of geometry matches my own geometric ontogeny very well, with the sequence: Euclidean, projective, transformational. But I doubt whether that ontogeny applies to the geometrical development of anyone in school today.

The work of Piaget and Garcia is carefully described at the beginning of chapter 5. But in the book as a whole, the song 'Ontogeny recapitulates phylogeny' is sung rather quietly. It might help to stop worrying whether the theory is true (it isn't), and instead ask whether it is a stimulating and fruitful idea. If we question whether historical investigations can enhance our awareness and especially our awareness of our own psychological development, the answer from this book is a resounding and unequivocal 'yes'. Particularly when modern psychological research has reached an *impasse* in its analysis of conceptual development, historical investigations provide the results of a longitudinal study.

The richness of this field is indicated by the bibliographies. Forty-seven sections of the book each have their own bibliography. Chapter 7 has an eight page bibliography. Chapter 11 consists of a forty-eight page annotated bibliography subdivided by language; Chinese, Danish, Dutch, English, French, German, Greek and Italian. In the English section, *For the Learning of Mathematics* is cited 37 times and *ESM* but 3 times. We are clearly missing out on something!

If readers of *ESM* look at no other part of this book, they should read chapter 3. It is only 30 pages long and reports realistically and crisply on the need for and difficulties in doing research in this area. In this chapter, individual writers have been given their heads to report research; this chapter is not a committee report. In every case described, making pupils into better mathematicians is the object of the exercise.

Today's teachers are hard pressed, and I cannot imagine all teachers of mathematics making history their hobby, as many of these authors have done. It is the value of *today's* mathematics that makes the *history* of mathematics a viable area of study and the historians need to recognise their dependence. The historians could do the rest of us sterling service if they would write historical commentaries on otherwise non-historical maths courses.

In such a well-resourced book there are likely to be surprises for everyone. I will finish by mentioning three which made me think.

1. A disaffected group of teenagers in Argentina writing and performing a play about the death of Galois. (Chapter 10)
2. A study of traditional geometrical patterns in post-colonial Mozambique. (Chapter 4, especially page 108)
3. A private school in the USA taking history as the unifying theme for the whole curriculum. (Chapter 8)

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