MATHEMATICS ASSERTIVENESS COURSE

Contents

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Carolyn Kennett, 2000

1 The Whole Number System

In Western mathematics we use a system known as base 10. Thus we have 10 units equals one ten, 10 tens equals one hundred, ten hundreds equals one thousand and so on. However, we could use other bases. In fact computers use a base of 2 and many Aboriginal tribes tended to use a base of 5. In the Gomileroi counting system, the word for 4 is a compound of 'two and two'. There are far more complex Aboriginal number systems where seemingly unrelated words are used to count by ones up to 50. As there are also words for 100, 500 and 1000, this language may represent a number system with a base of 50. Extremely difficult to learn by anyone's standards! The early European anthropologists visited Aboriginal tribes briefly, with preconceived ideas of their numeracy needs and abilities. Thus, often quoted in school books of the 50's and 60's is the inaccurate account of 'one, two, many' as the way Aboriginal people counted. Like the Europeans, Aborigines did not always need to be accurate. If there is a crowd we say there are 'lots of' or 'hundreds of' or 'thousands of' people much the same as the Aborigines used 'small mob' or 'large mob'.

Our Decimal System

In your groups you will reacquaint yourselves with the decimal system using the base 10 blocks. Note that the decision to use a base of 10 was an arbitrary choice made by the European man sometime in the past based on 10 fingers. It is not something carved in stone and handed down from above. Indeed other ancient cultures used different bases, for example the Babylonians. For this session we will be making use of the Dienes, base 10, block material. We could also use some basic units of currency to do these exercises. Can you suggest which units we would need?

You will notice that there are pieces of wood of different sizes.

Suppose we let the very **small cube** equal 1. How many small cubes make one **rod**? __________

Thus, if we let the small cube equal 1 we should let the **rod** equal 10. Now look at the flat pieces. How many rods make one **flat**? __________

How many small cubes make one **flat**? __________

Now look at the **large cube**. How many flats make a large cube? __________

How many rods make a large cube? __________

How many small cubes make a large cube? __________

Suppose instead we gave the small cube a monetary value of **1 c**. What monetary value would the rod have? __________

What would the flat be worth? __________

What would the large cube be worth? __________

Now suppose we let the small cube equal **1 cubic cm**. What would the volume of the rod be? __________

What would the volume of the flat be? __________

How many small cubes will fit inside one metre cube? __________

Using the Diennes Blocks

Using the blocks, build the numbers 425 and 2782.

1. (a) How many 10's (i.e. rods) in the model of 425? ________________
   (b) If we replaced the flats with rods, how many 10's are there in 425? ____
   (c) How many 10's in the model of 2782? ________________
   (d) If we replaced the large cubes and the flats with rods, how many 10's are there in 2782? ________________

2. (a) How many 100's (i.e.flats) in the model of 425? ________________
   (b) How many 100's in the model of 2782? ________________
   (c) If we replaced the large cubes with flats, how many 100's are there in 2782? ________________
   (d) How many 1000's (i.e. large cubes) in 2782? ________________

   In each case relate your result to the original number

3. Use the blocks to add 425 and 2782.

4. Use the blocks to subtract 38 from 126.

5. There are a lot of mam-made conventions in mathematics. some of them are to enable us to condense information, one example is the use of exponential form. Look for a pattern then complete the table below.

$10 = 10 = 10^1$	$30 = 3 \times 10 = 3 \times 10^1$
$100 = 10 \times 10 = 10^2$	$200 = 2 \times 100 = 2 \times 10^2$
$1000 = 10 \times 10 \times 10 = 10^3$	$6000 = 6 \times 1000 = 6 \times 10^3$
$\_\_\_\_ = 10 \times 10 \times 10 \times 10 = 10^4$	$70000 = \_\_\_ \times 10000 = 7 \times 10^{\_\_}$
$100000 = \_\_\_\_\_\_\_\_ = 10^5$	$\_\_\_\_ = 9 \times 100000 = 9 \times \_\_\_\_$
$1000000 = 10^{\_\_}$	$8000000 = \_\_\_\_\_\_\_\_ = 8 \times 10^6$

Large numbers are often expressed in exponential form, especially when using a scientific calculator. Some earlier models of calculators will show the number 50000000 as 5   07 but later models will show the correct notation of 5×10^7.

6. Express the following as a single number

$5 \times 10^5 + 4 \times 10^4 + 3 \times 10^3 + 8 \times 10^2 + 6 \times 10^1 + 2 =$
$4 \times 10^5 + 0 \times 10^4 + 3 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 8 =$
$3 \times 10^6 + 9 \times 10^5 + 2 \times 10^1 + 5 =$
$4 \times 10^4 + 8 \times 10^1 =$

Can you see a relationship between the multiple of 10^X and its place value?

Exploring the 9 Times Tables

Activity 1

1. Write down your birthday (Carolyn 24 07 62)
2. Rewrite these numbers in any order (762240)
3. Now subtract the smallest number from the largest number (762240 - 240762 = 521478)
4. Add the digits in your answer together (5+2+1+4+7+8 = 27)
5. Keep adding the digits together until you are left with one number (2+7 = 9). Is your number 9 too?

Activity 2

1. Write down the first 15 numbers in your 9 times tables.
2. In pairs discover one or two interesting things about these numbers.
3. Using our hands we can find the answer to the nine times tables from 9×1 to 9×10.
4. Circle those numbers that are divisible by 9.

   451 351 476 675 467 433 2 544 671

2 Order of Operations

Remember that we said that the brackets were very important when doing Exercise 1 on the calculator. This is because brackets have a higher priority than addition.

Consider the following situation. You have worked from 2 p.m. to 9 p.m. on a major proposal. As you were required to complete it by the following day you can claim overtime. The rates from 9 a.m. to 5 p.m. are \$25 per hour and from 5 p.m. to midnight rise to \$37.50 per hour.

Mathematically, this can be expressed as $3 \times \$25 + 4 \times \$37.50$. How much do you think you earned? Which attempt below is most reasonable?

Attempt 1	**Attempt 2**
$3 \times 25 + 4 \times 37.50$	$3 \times 25 + 4 \times 37.50$
$= 75 + 4 \times 37.50$	$= (3 \times 25) + (4 \times 37.50)$
$= 79 \times 37.50$	$= 75 + 150$
$= 2962.50$	$= 225$

Mathematical expressions can be read by anyone regardless of their spoken language. In order to avoid confusion certain conventions (or accepted methods) must be followed. One of these is the order in which we carry out arithmetic.

Brackets:	Evaluate the expression inside the brackets first eg. $(3+5) = 8$
Indices:	Evaluate the expressions raised to a power eg. $(3+5)^2 = 8^2 = 64$
Division:	Divide or multiply in order from left to right.
Multiplication:	eg. $6 \times 5 \div 3 = 30 \div 3 = 10$
Addition:	Add or subtract in order from left to right.
Subtraction:	eg. $4 + 5 - 2 + 4 = 9 - 2 + 4 = 7 + 4 = 11$

This can easily be remembered by the acronym **BIDMAS**. Try the following exercise:

(a) $6 + (7 - 4) =$

(b) $4 \times (8 + 3) =$

(c) $(5 + 3) \times (9 - 4) =$

(d) $6 + 7 \times (5 - 3) =$

(e) $12 - 8 \div 4 + 6 \times 3 =$

(f) $3^2 \times 4 + (3 \times 2 + 4) \div 5 =$

(g) $6 \times (3 + 2 - 8 \div 2)^2 + 4 =$

(h) $4 + 5 \times 6 \div (6 + 4) =$

Modern calculators have been programmed to observe the order of operations and also come equipped with a bracket function. You just enter the brackets in the appropriate spots as you enter the calculation. Some older calculators with a bracket function will tell you how many brackets you have opened, they do this by having on the display **(01** or **[01**.

Check all your answers to the above exercise with your calculator (here are the questions again).

(a) $6 + (7 - 4) =$

(b) $4 \times (8 + 3) =$

(c) $(5 + 3) \times (9 - 4) =$

(d) $6 + 7 \times (5 - 3) =$

(e) $12 - 8 \div 4 + 6 \times 3 =$

(f) $3^2 \times 4 + (3 \times 2 + 4) \div 5 =$

(g) $6 \times (3 + 2 - 8 \div 2)^2 + 4 =$

(h) $4 + 5 \times 6 \div (6 + 4) =$

3 Distributive Law

Suppose you have four children and each child requires a pencil case (\$2.50), a ruler (\$1.25), an exercise book (\$2.25) and a set of coloured pencils (\$12) for school.

One way we can calculate the cost is by multiplying each item by 4 and adding the result:

$$(4 \times \$2.50) + (4 \times \$1.25) + (4 \times \$2.25) + (4 \times \$12) = \$10 + \$5 + \$9 + \$48 = \$72$$

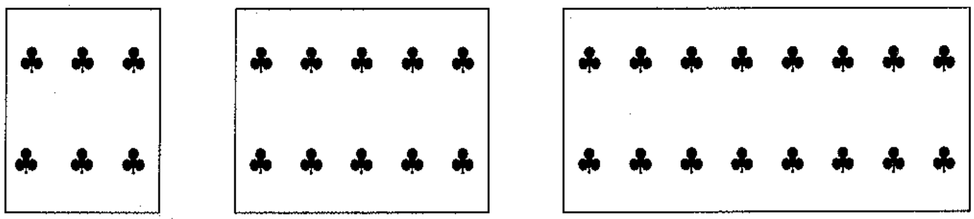
This took quite a bit of effort so perhaps there might be a simpler method. Why not work out how much it will cost for one child and then multiply by 4?

$$(\$2.50 + \$1.25 + \$2.25 + \$12) \times 4 = \$18 \times 4 = \$72$$

Let's simplify the numbers a little to shed light on this simpler method.

$$(2 \times 3) + (2 \times 5) = 2 \times (3 + 5)$$

$$2 \times 3 \quad + \quad 2 \times 5 \quad = \quad 2 \times 8$$



$$(2 \times 3) \quad + \quad (2 \times 5) \quad = \quad 2 \times (3 + 5)$$

Therefore when removing the brackets from an expression such as $2 \times (3+5)$ you must multiply every term inside the bracket by the number outside the bracket.i.e.

$$2 \times (3 + 5) = 2 \times 3 + 2 \times 5$$

To simpify the expression even further mathematicians omit the multiplication sign between the 2 and the bracket. Therefore $2 \times (3 + 5)$ would read $2(3 + 5)$.

Try some for yourself:

Regroup the following as in the example:

eg. $(4 \times 13) + (4 \times 7) = 4 \times (13 + 7)$
(a) $(3 \times 2) + (3 \times 5) =$
(b) $(5 \times 4) + (5 \times 2) =$
(c) $(7 \times 93) + (7 \times 7) =$
(d) $(2 \times 25) + (25 \times 48) =$

Remove the bracket as in the example:

eg. $4 \times (13 + 7) = (4 \times 13) + (4 \times 7)$
(a) $3 \times (2 + 5) =$
(b) $5 \times (4 + 2) =$
(c) $7 \times (93 + 7) =$
(d) $25 \times (2 + 48) =$

Exercise

Regroup the following and then calculate the answer

(a) $(2 \times 13) + (2 \times 7)$
(b) $(15 \times 8) + (15 \times 2)$
(c) $(8 \times 43) + (8 \times 7)$
(d) $(23 \times 96) + (23 \times 4)$
(e) $(5 \times 13) + (5 \times 7)$

4 Calculator Skills

Getting to know your calculator

Notice that the keys on the calculator have one character written on them and another written above them. You will find out about some of the different characters in this session and in the next block.

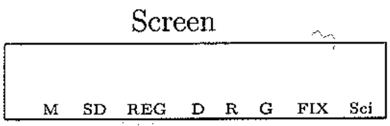
Exercise 1

Enter any four digits into your calculator. Then without pressing any other buttons enter the same four digits again. Divide the eight digit number by 73. Divide the result by 137. What is the result? Try it again beginning with another 3 digits. Did you get the answer you expected? Why do you think this works?

Most of this section refers to the Casio brand of calculators predating the fx-82 model. Much of the information carries over to other brands and where possible I have included information for different brands. Most of the differences will appear when we are accessing the statistics mode in the calculator.

Firstly we will familiarise ourselves with the calculator. Find the key with SHIFT written on it or above it. On some calculators this button will have 2nd Function written on or above it instead. This key accesses the functions written above the keys, as opposed to the functions written on the keys. Find the key marked MODE, this is a seperate key for many calculators but some people may need to use the SHIFT key to access the mode key. The MODE key lets us determine what mode the calculator is operating in. There are many choices other than the normal computational mode that is the default mode for your calculator. We will discuss various modes as we work through this material. For now we will stay in normal mode. To check that you are in normal mode you should look at the screen of your calculator for the small letters at the bottom.

You may have any of the following appearing on the bottom of your screen. For what we are about to do you want only one of D,R or G and possibly an M.



You will always have a D,G or R appearing on your calculator screen. This tells you what measurement you are using when you put in angles and use them to do calculations. Since you are mostly Statistics students we are not going to bother with this stuff. The M indicates that you have something in memory. The other letters indicate that your calculator is working in a particular mode. SD is Statistics mode, REG (on Sharps these appear respectively as Stat$x$ and Statxy) indicates that you are in regression mode. The letters FIX and Sci stand for a fixed number of decimal places and scientific notation respectively. We want to be in normal mode for this first section so you should find the mode button (it is near the top of the calculator on Casios and appears as a second function on Sharps). Choose mode NORM or

COMP to get out of the SD and REG modes. Choose NORM to get out of the FIX and Sci modes. On the Casios you will see the following screens

Enter		Screen
[MODE]	Choose 1 to get out of SD and REG modes	COMP 1 · SD 2 · REG 3

Enter		Screen
[MODE][MODE]	You can make any choice here	Deg 1 · Rad 2 · Gra 3

Enter		Screen
[MODE][MODE][MODE]	Choose 3 and then 1 here to get out of Fix and Sci mode	Fix 1 · Sci 2 · Norm 3

Hopefully you now know how to get into normal computational mode. For the next section we will stay in this mode.

Order of Operations

Modern calculators have been programmed to observe the correct order of operations. For example:

Enter	Screen
[4][+][3][×][2][=]	$4 + 3 \times 2$ 10

This means you can enter long, complicated expressions and the calculator will work out what should be done first. If your calculator does not give you the answer 10 above then it does not follow the order of operations and you will need to be careful when calculating an expression with many parts to it. To enter a negative number in the calculator you do one of two things depending on which calculator you have. Look for a button marked [(−)] or [+/−]. If you have the button marked [(−)] then to enter a negative number press [(−)] and then the number. For example to enter −5 press [(−)][5]. (If your calculator has the [+/−] button you first enter the number and then press [+/−]. So to enter −5 you would press [5][+/−].) For example, if we wanted to calculate -4×-5 we would do the following:

Enter	Screen
(−) 4 × (−) 5 =	-4×-5 20

Bracket Buttons

Most modern calculators also come equipped with a bracket function. Find the buttons marked (and). These open and close brackets and you just enter the brackets in the appropriate spots as you enter the calculation. Suppose we wanted to calculate $4 \times (3+2)^2 \div 20$, then

Enter	Screen
4 × (3 + 2) x^2 ÷ 2 0 =	$4 \times (3+2)^2 \div 20$ 5

Note that sometimes we can mislead our calculators into doing the wrong thing. In these expressions

$$\frac{4+2}{3}\quad,\quad \frac{4}{2+2}\quad,\quad \sqrt{4+8}\quad,$$

there are brackets implied even though they are not written. So we need to put them in when we enter them into the calculator So we enter

$$\frac{(4+2)}{3}\quad,\quad \frac{4}{(2+2)}\quad,\quad \sqrt{(4+8)}\quad,$$

Let's do the last one on the calculator

Enter	Screen
√ (4 + 8) =	$\sqrt{(4+8)}$ 3.464101615

Replay

Sometimes in Statistics you will need to do a calculation like $4.35 \pm 1.96 \times 2.1432$. Recall that the $\pm$ in the expression means you want to calculate both the expression with a plus and the one with a minus. You can use the replay button to make this easier.

Enter

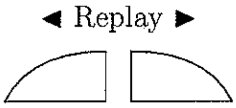
[4][·][3][5][+][1][·][9][6][×][2][·][1][4][3][2][=]

Screen

4.35 + 1.96 × 2.1 –►
8.550672

The replay buttons are at the top of the panel on the front of your calculator.

Press the button on the left →



Notice the screen now has a flashing line at the end of the expression

Screen ↓

◄– 1.96 × 2.1432_
8.550672

As you push the left button the line moves along the expression and whatever it is under flashes. Move it until it is under the +.

↓ Screen

The plus sign + will be flashing

4.35+1.96 × 2.1 –►
8.550672

Now push the [–] button. The screen now has a minus instead of a plus. Push [=] and you have calculated $4.35 - 1.96 \times 2.1432 = 0.149328$.

You can also use these buttons to insert or delete things in the expression. For example, suppose I want to calculate $\sqrt{9+16}$ and I do the following:

Enter

[√][9][+][1][6]

Screen

√9 + 16

and then I realise that I forgot to put in the brackets. Use the replay buttons until you are under the 9 and then use the delete and insert functions. these functions may be combined in one button or on seperate buttons. On the Casio calculators the button is red, next to the ON button and has DEL written on it and INS above it. To insert a bracket before the 9 we move the cursor as described above and then do the following:

Enter	Screen
INS [SHIFT] [DEL] [( ]	$\sqrt{}$ (9 + 16

Use the replay button (right) to get to the end of the expression and put in another bracket. Enter

Enter	Screen
[ )] [=]	$\sqrt{}$ (9 + 16) 5

Fractions

To enter a fraction find the button [$a\frac{b}{c}$]. To enter $\frac{4}{5}$ and $2\frac{1}{2}$:

Enter	Screen
[4] [$a\frac{b}{c}$] [5]	4 ⌟ 5 0

Enter	Screen
[2] [$a\frac{b}{c}$] [1] [$a\frac{b}{c}$] [2]	2 ⌟ 1 ⌟ 2 0

At this point you have entered $2\frac{1}{2}$. Now lets do some more things with this number.

Enter	Screen
[=] to get the screen	2 ⌟ 1 ⌟ 2 2 ⌟ 1 ⌟ 2

To change $2\frac{1}{2}$ from a mixed number to an improper fraction;

Enter	Screen
[SHIFT] [$a\frac{b}{c}$] to get the screen	2 ⌟ 1 ⌟ 2 5 ⌟ 2

Enter		Screen
$a\frac{b}{c}$	to get the screen	2 ⌟ 1 ⌟ 2 2.5

Now we get the decimal equivalent of $2\frac{1}{2}$, that is 2.5. Enter $a\frac{b}{c}$ again and we get back to the mixed number form of the fraction.

Enter		Screen
$a\frac{b}{c}$	to get the screen	2 ⌟ 1 ⌟ 2 2 ⌟ 1 ⌟ 2

Try this one:

Enter	Screen
4 $a\frac{b}{c}$ 5 =	4 ⌟ 5 4 ⌟ 5

Enter	Screen
$a\frac{b}{c}$	4 ⌟ 5 0.8

Enter	Screen
$a\frac{b}{c}$	4 ⌟ 5 4 ⌟ 5

Scientific Notation

Do this calculation

Enter	Screen
1 ÷ 9 8 7 6 5 4 3 2 1 =	1 ÷ 987654321 1.0125^{-09}

Notice the little −09 in the corner of the bottom part of the screen. This means that the answer to this calculation is 1.0125×10^{-9}. This says that the number is really 1.0125 with the decimal place shifted 9 places to the left, i.e

$$0.0000000010125$$

Now do the calculation 98765×987654321 to get the answer 9.754567901^{13} on your screen. The little 13 tells you that the answer is 9.75456790 with the decimal place shifted 13 places to the right, i.e.

$$97545679010000$$

Keep an eye out for those little numbers!!

Memory

You have 10 memory slots in your calculator if you have a Casio fx-82 TL. They are called A,B,C,D,E,F,X,Y,M and ANS.

A,B,C,D,E,F,X and Y all work the same way when you are in Comp mode as we are in now. They are accessed by using the [STO] and [RCL] buttons in combination with the letters which are written above some of the keys on the upper section of your calculator in a different colour font than the functions accessed by the [SHIFT] key. To put someting into one of these memories you first need to have it as an answer on the screen. For example:

Enter	Screen
[4] [×] [6] [=]	4 × 6 24

To store in memory A.

Enter	Screen
[STO] [(−)] (A)	A= 24

and 24 will be placed in A. To recall it use;

Enter	Screen
[RCL] [(−)] (A)	A= 24

To use it in an equation you don't need to recall it you can tell the calculator to use whatever is in the particular memory slot you want to use by using the key with ALPHA written above it. So suppose we now wanted to do 120×24 where 24 is the number in A.

Enter	Screen
[1] [2] [0] [×] [ALPHA] [(−)] (A) [=]	120× A 2880

Memory slot M works a little differently. You can do all the same things with M that we just mentioned for A but a bit more as well. Find the button above the AC button that looks like this:

M− M

[M+]

DT ⌞CL⌟

If you press [M+] it will add whatever is on the screen to whatever is already in M. If you press [SHIFT] [M+] it will activate the M− and subtract what is on the screen from whatever is already in the memory M. To determine if M already has something in it look for a small M on your screen.

Screen
M 0

Enter		Screen
[SHIFT] [AC/ON] (MCl SCl) [=]	will clear the memory M	MCl 0

Finally the memory slot [ANS]. This is a very short term memory slot. It has in it the last thing shown as an answer on the bottom line of the screen.

Enter	Screen
[1] [$a\frac{b}{c}$] [1] [0] [+] [1] [$a\frac{b}{c}$] [1] [2] [=]	1⌟10 + 1⌟12 11⌟60

The number $\frac{11}{60}$ is now in memory ANS. By the way the calculation I just did was $\frac{1}{10} + \frac{1}{12}$. Suppose the sum I wanted to do now was $\sqrt{\frac{1}{10} + \frac{1}{12}}$. I can do this by doing the following

Enter	Screen
[$\sqrt{}$] [ANS] [=]	$\sqrt{}$ANS 0.428174419

Once I press the [=] button the new answer will get put into the ANS memory.

Exercise 2

Calculate $(43 + 29) \div (29 - 17)$ using your calculator in the following way. First work out the denominator i.e. $29 - 17 =$ and then press **M+** or $\mathbf{M_{in}}$. Now evaluate the numerator $43 + 29 =$ and divide the result by the number held in memory by pressing $\div$ and then **RM** or **MR** and finally press =.

More modern calculators can do this sum in a more simple way using the bracket function on the calculator (look for buttons with (and) on them). If you have the bracket functions you can enter the sum as you read it i.e. $(43+29) \div (29-17)$ and then push = to get the answer. The brackets in this expression are very important as we will learn in the next section of work.

Exercise 3 - The Target Game

Find a partner and your calculator. If you can, change the mode of your calculator to give you no decimal places. If you don't know how to do this, ask your teacher.
Player 1 should enter any number into the calculator.
Player 2 then has to multiply this number by another number so that the answer will be as near to the target number of 100 as possible.
Player 1 then multiplies this answer by another number trying to get nearer still to 100.
Take turns until one player 'hits' the target by getting 100 on the calculator display.
Record your game below.
Try hitting a different target.

5 Estimation

Since it is easy to make a mistake when using your calculator, get into the habit of estimating what the answer should be and comparing the calculator answer with your estimate.

Estimate solutions to each of the following **without** using a calculator.

(a) $59 + 62$
(b) $954 + 215$
(c) $1029 - 287$
(d) 98×12
(e) $259 \div 49$
(f) $116 + 490 + 810$
(g) 298×11
(h) 9×31
(i) $53 + 53 \times 98$
(j) $895 \div 287$

Without using your calculator circle the nearest correct answer for each of the following

(a) 326×153
(i) 4988
(ii) 49878
(iii) 498708

(b) 13.456×298
(i) 300
(ii) 3000
(iv) 30000

Rounding Off

1. Calculate the value of the following (to two decimal places). Your calculator may be able to do this for you, check your manual. Otherwise, rounding to two decimal places is like rounding to the nearest ten. Look at the first three numbers after the decimal place, eg. in 45.8739 I would look at 873. I then round this number to the nearest ten, which in the example is 870 since 873 is closer to 870 than to 880. My number rounded to two decimal places becomes 45.87 (Note: I have replaced the numbers in the decimal places with the first two of my rounded number. Try these exercises:

   (a) 3.5678
   (b) 245.867
   (c) 23.0098
   (d) 237.043
   (e) 0.263

2. Round off the following to the nearest whole number; (you can think of this as rouding to the nearest dollar if you wish)

   (a) 267.8

   (b) 25.143

   (c) 0.345

   (d) 0.71

   (e) 79.5

3. Name two numbers larger than 10 that multiplied together give 540. Explain how you solved this problem.

Improving Estimation Skills

Signs

The most commonly used mathematical signs are as follows:

Sign	Meaning
$+$	add
$\times$	multiply
$=$	equals
$>$	greater than
$\geq$	greater than or equal to
$\approx$	approximately equal to
$\surd$	square root

Sign	Meaning
$-$	minus or subtract
$\div$	divide
^{2}	square
$<$	less than
$\leq$	less than or equal to
$\neq$	not equal to
$\therefore$	therefore

Exercise 1
Use a $+$ or $-$ or $\times$ or $\div$ and $=$ to complete the following number sentences:

1. 4 __ 6 __ 10

2. 24 __ 6 __ 4

3. 6 __ 7 __ 42

4. 42 __ 7 __ 35

Exercise 2

Fill in the blank space with a $>$ or $<$ sign so that the number sentence is sensible.

1. 324 ____ 331
2. 4×8 ____ 3×9
3. $4 + 10 \div 2$ ____ $2^2 + 16 \div 4$

Exercise 3

Change the left hand side of each equation by adding a $\surd$ or a power such as ^{2} so that the two sides of the equation are equal.

1. $16 = 4$
2. $5 = 25$
3. $3 = 9$
4. $64 = 8$

Exercise 4

State two values we can use instead of the letter a so that the statement is true.

1. $2a \geq 13$
2. $2a + 5 \geq 11$
3. $2(a + 3) \leq 10$
4. $\sqrt{a} \leq 4$

Estimation Skills I

When we go shopping we usually have some idea of how much to put in the trolley so that we do not embarrass ourselves at the checkout by not having enough money. What we are in fact doing, as we drive the trolley up and down the aisles, is estimating. As very few people take a calculator into the store, mental arithmetic plays a crucial role in estimating the cost.

Exercise 5

1. For each of the monetary amounts below round to the nearest dollar. That is, if the digit after the decimal point is ≥ 5 then increase the dollar amount by 1 and replace all the digits after the decimal point with zeros. If the digit immediately after the decimal point is < 5, leave the dollar amount the same and replace all the digits after the decimal point with zeros. For example;

   \$2.05 is close to \$2.00, \$6.85 is close to \$7.00 and \$1.35 is close to \$1.00.

   So,
   $$\$2.05 + \$6.85 + \$1.35 \approx \$2.00 + \$7.00 + \$1.00 = \$10.00$$

   Using these estimation skills determine which of the shopping lists below you are able to afford if you only have \$20.

   (a)
   1 dozen eggs @\$2.05
   2 litres oil for \$4.94
   1 roast leg for \$7.95
   1 can apple juice for \$1.65
   1 magazine for \$2.10

   (b)
   3 packs of choc-chips @ \$2.05
   1 carton of yogurt @ \$1.85
   6 packets of fruit medley @ \$1.85 each

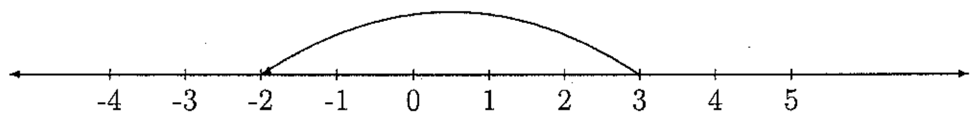
   (c)
   3 dozen eggs @\$1.85 per dozen
   500g coffee for \$5.85
   250g bacon @ \$7.95 kg

   (d)
   10 cans Pal dog food @ \$1.25 each
   5 cans pineapple pieces @ \$0.85
   2 boxes sandwich bags @ \$2.25 each
   2 bottles wine @ \$3.95 per bottle

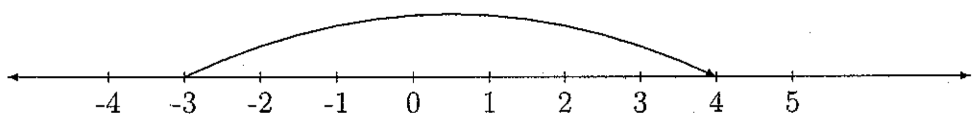
2. Now check your estimates above by adding up each shopping list

6 Negative Numbers

So far we have examined the numbers which extend beyond zero. However, we do not always use positive numbers. For example, temperature can be negative as can your bank account balance. Look at the diagram below which we will refer to as the number line. When we subtract a positive number (or add a negative number) we move the pointer to the left to get a smaller number. Represented below is the sum $3 - 5 = -2$.



Similarly, when we add a positive number (or subtract a negative number) we move the pointer to the right to get a larger number. The diagram below represents $-3 + 7 = 4$.



Card Game

Each player has ten black and ten red cards. Let the black cards represent positive numbers and the red cards represent negative numbers. Start on Zero. Shuffle the cards. Draw the first card and move the rubber band to the left if negative and the right if positive. Repeat this process until all of the cards have been drawn. The players to reach zero when all the cards have been used are the winners.

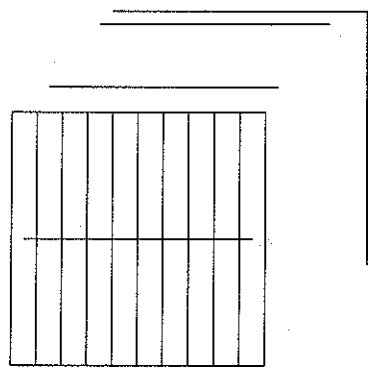
7 Decimals or The Numbers in-between Whole Numbers

Most of us use decimals every day when dealing with money. Since 100 cents is equal to one dollar, then one cent is one hundredth of a dollar. This is expressed as $).01. Similarly, we need ten 10c pieces to make one dollar, so 10c = $0.10.

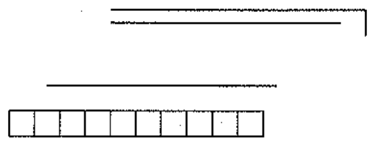
If we were to add 56c to 70c, none of us would have trouble figuring out the answer of $1.26. So $0.56 + 0.7$ is just as easy. When working with decimals, try to think of the numbers in terms of dollars and cents so that the question is easier to understand.

We are going to use the base ten blocks to model decimals. Suppose we were to let the large wooden cube equal 1. We would then find that 10 of the flats would make 1. Thus, each flat equals $\frac{1}{10}$ or 0.1 of the large cube.

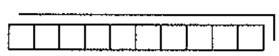
Similarly, each rod equals $\frac{1}{100}$ or 0.01 of the wooden cube and each small cube equals $\frac{1}{1000}$ or 0.001 of the large wooden cube.



= 1.0



= 0.1



= 0.01

= 0.001

1. Use these meanings of the blocks to represent

   (a) 0.186

   (b) 0.309

   (c) .008

i. How many small cubes in the model of .186? ______

ii. If we replaced all the blocks with the corresponding number of small cubes, how many small cubes ($\frac{1}{1000}$) would there be in .186? (this is the same as dividing 0.186 by 0.001) ______

iii. How many rods in the model of 0.180? ______

iv. If we replaced all the blocks with rods, how many rods ($\frac{1}{100}$)would we have? (this is the same as dividing 0.180 by 0.01) ______

v. How many flats ($\frac{1}{10}$)in 0.100? ______

2. Use the blocks to

(a) add 0.098 and 0.209

(b) subtract 0.187 from 0.309

3. Use the blocks to represent each of the following

(a) 0.05

(b) 0.154

(c) 0.039

(d) 0.005

(e) 0.111

Arrange these numbers now from smallest to largest. What is the middle value of this rearranged set? ______

4. To find one tenth of a number, you could simply exchange large cubes for flats, flats for rods, rods for small cubes. Since you have found $\frac{1}{10}$th of every part of the number you have the answer.

(a) Find one tenth of the following

i. 3.45 ______

ii. 1.09 ______

iii. 0.6 ______

iv. 0.78 ______

v. 0.09 ______

Similarly, to find one hundredth of a number, replace each part with one hundredth of its value. For example, replace 3 flats with 3 small cubes.

(b) Find one hundredth of each of the following (you could think in terms of dollars and cents)

i. 3.4 ______

ii. 0.9 ______

iii. 180 __________

iv. 2.5 __________

v. 43 __________

(c) Can you find one thousandth of the following?

i. 4 __________

ii. 1 __________

iii. 23 __________

iv. 12 __________

v. 47 __________

5. When we divide by ten it is the same as multiplying by $\frac{1}{10}$th. To find one thousandth of a thing is the same as dividing by one thousand. Fill in the table below using this information:

$$2460 \div 10 = 246$$

$$2460 \div 100 = 24.6 = 246 \div 10$$

$$2460 \div 1000 = 2.46 = 24.6 \div 10$$

Whole Numbers				Proper Fractions			
1000's	100's	10's	Ones	1/10's	1/100's	1/1000's	
2	4	6	0				
							$\div$10 or $\times$1/10
							$\div$100 or $\times$1/100
							$\div$1000 or $\times$1/1000

Can you see a quick way to divide by 10's, 100's and 1000's?

6. In the same way as we could expand the whole numbers using powers of ten we can also expand the fractional numbers using negative powers of ten. Exponential notation can also be used for decimals. So that if we can write 4000 as 4×10^3 we can write 0.002 as 2×10^{-3}. Our short hand notation allows us to write the following:

   $10 = 10^1$ $\quad\quad$ $\frac{1}{10} = 10^{-1}$

   $100 = 10^2$ $\quad\quad$ $\frac{1}{100} = 10^{-2}$

   $1000 = 10^3$ $\quad\quad$ $\frac{1}{1000} = 10^{-3}$

   Complete the following table. Think carefully about what the value of 10^0 might be.

10^{-4}	10^{-3}		10^{-1}	10^0		10^2	
$\frac{1}{10000}$		$\frac{1}{100}$			10		1000

7. Given that $45 \times 10^{-2} = 0.45$ and $35000 \times 10^{-4} = 3.5$, try to work out the following with or without your calculator.

   $3.56 \times 10^{-1} =$ _________ $\quad\quad$ $362 \times 10^{-4} =$ _________

   $872 \times 10^{-2} =$ _________ $\quad\quad$ $3.172 \times 10^{-3} =$ _________

   $326 \times 10^{-3} =$ _________ $\quad\quad$ $48706 \times 10^{-6} =$ _________

   $27.8 \times 10-2 =$ _________ $\quad\quad$ $2.335 \times 10^{-8} =$ _________

8. Use your calculator to square the following by entering the number and then pressing x^2. The answers may be displayed in exponential form i.e. they may look like 4.678×10^8 or 4.678^{08}. If you get either of these displays then the answer is 4.678×10^8 which equals 467 800 000.

   $12^2 =$ _________ $\quad\quad$ $123^2 =$ _________

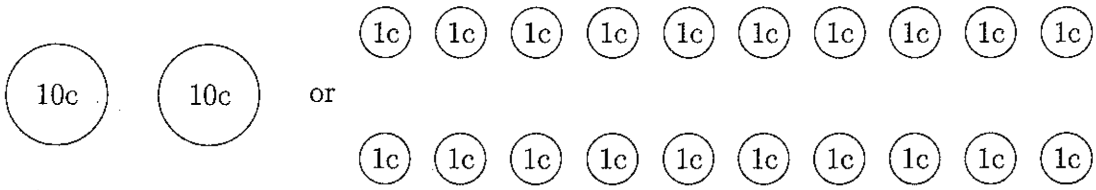
   $1234^2 =$ _________ $\quad\quad$ $12345^2 =$ _________

   $123456^2 =$ _________ $\quad\quad$ $1234567^2 =$ _________

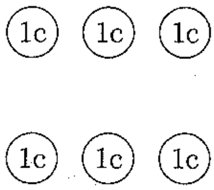
Multiplication of decimals

We want to try to model the multipliaction of decimals using money as our concrete material. Let's try to work out what 0.3×0.2 is using money.

0.2 of one dollar is 20 cents.



Therefore, 0.3 of 0.2 (or 3 tenths of 20 cents) would be the first 3 coins in each row of 10 one cent coins. Therefore $0.3 \times 0.2 = 0.06$.



As shown above use money to make physical models of these algorithms.

2×3 0.2×3 3×0.2 0.2×0.3

Discuss your results and your representations with other members of your group. Can you think of a quick way to multiply decimals? Look for a pattern when using your calculator to solve the following:

$300 \times 4 =$ ________	$500 \times 7 =$ ________	$6 \times 0.2 =$ ________
$30 \times 4 =$ ________	$50 \times 7 =$ ________	$0.6 \times 0.2 =$ ________
$3 \times 4 =$ ________	$5 \times 7 =$ ________	$0.06 \times 0.2 =$ ________
$0.3 \times 4 =$ ________	$0.5 \times 7 =$ ________	$0.006 \times 0.2 =$ ________
$0.03 \times 4 =$ ________	$0.05 \times 7 =$ ________	$0.0006 \times 0.2 =$ ________
$0.003 \times 4 =$ ________	$0.005 \times 7 =$ ________	$0.0006 \times 0.02 =$ ________

Can you find any rule for the multiplication of decimals?

Estimation Skills II

Usually when we make an educated guess or estimate, we round the numbers involved off to the nearest leading digit. It is just a matter of multiplying the non-zero digits together and multiplying the powers of ten together.

For example:

If we were to multiply 345 by 89 it is good practice to estimate first to be more certain of our solution.

Step 1: Round 345 to the nearest hundred:	300
Step 2: Round 89 to the nearest ten:	90
Step 3: Multiply the leading digits together:	$3 \times 9 = 27$
Step 4: Multiply the powers of ten together:	$100 \times 10 = 1000$
Step 5: Complete the multipliaction:	$27 \times 1000 = 27000$
Finally we have:	$345 \times 89 \approx 300 \times 90 = 27000$

Exercise 1

Estimate each of the following and then check that your estimate is close to the actual answer by using your calculator.

1. 487×21
2. 8976×427
3. 6298324×4741
4. 0.812×28
5. 0.567×0.23
6. 0.25×7.98
7. 45.89×5.09
8. 29876×3.45

It is good practice to estimate before you calculate so that you have a good idea of what to expect.

Multiplication by powers of 10

Our number system increases by powers of 10 as we move to the left and decreases by powers of 10 as we move to the right. Given the number **123 456.78**, the **1** tells us how many 100,000's there are, the **4** indicates the number of 100's and the **7** indicates the number of tenths. Similarly when we multiply two numbers together such as 200×3000 we can use our knowledge of the place value system to represent this as $(2 \times 100) \times (3 \times 1000)$. Since the order is not important in multiplication (i.e. $2 \times 3 \times 4 = 4 \times 3 \times 2$) we can rewrite this as:

$$\begin{aligned} 200 \times 3000 &= (2 \times 100) \times (3 \times 1000) \\ &= (2 \times 3) \times (100 \times 1000) \\ &= 6 \times 100000 \\ &= 600000 \end{aligned}$$

Similarly we can calculate:

$$\begin{aligned} 4000 \times 0.05 &= (4 \times 1000) \times (5 \times \frac{1}{100}) \\ &= (4 \times 5) \times 1000 \times \frac{1}{100} \\ &= (20 \times 10) \\ &= 200 \end{aligned}$$

Exercise 2

Set your working out as shown above especially when decimals are involved.

(i) 30×600

(ii) 400×80000

(iii) 500×0.03

(iv) 700×0.03

(v) 2000×0.6

(vi) 0.8×0.3

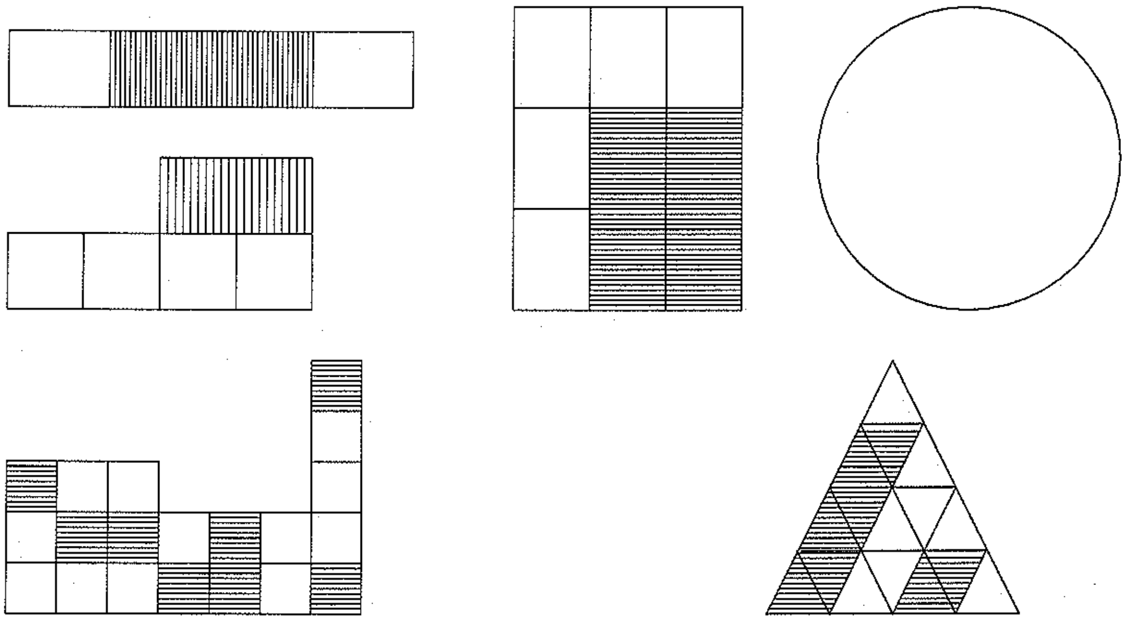
(vii) 0.03×0.09

(viii) 60×0.4

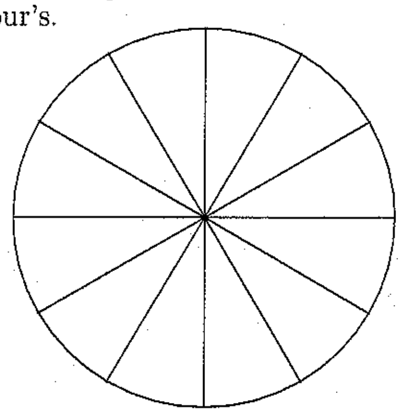
8 Fractions and Percentages

Exercise 1

In the figures below, what fraction is the shaded area of the whole figure?



Shade $\frac{2}{3}$ (two thirds) of the figure below. Compare your shading with your neighbour's.



You should have found from the exercise above that $\frac{2}{3}$ was equivalent to $\frac{8}{12}$.

Exercise 2

For the second activity we will do you should work in twos or threes. You will need a set of strips of paper.

Using the strip marked as $\frac{1}{2}$ for comparison, find other numbers of strips that make the same size as $\frac{1}{2}$.

Now do the same thing for $\frac{2}{3}$ and $\frac{3}{4}$.

Record all the different combinations of strips that you found that measured the same as

1. $\frac{1}{2}$

2. $\frac{2}{3}$

3. $\frac{3}{4}$

Exercise 3

1. Can you find two equivalent fractions for each of the following?

$$\frac{1}{5} = \quad = \qquad \frac{3}{8} = \quad = \qquad \frac{2}{3} = \quad =$$

Compare your answers with the rest of your group. Try entering each of your answers into the calculator. Press =. What do you notice?

2. Use your calculator to simplify the following fractions

$$\frac{3}{9} = \frac{}{3} \qquad \frac{25}{100} = \frac{5}{} \qquad \frac{21}{28} = \frac{}{4}$$

$$\frac{15}{25} = \frac{}{5} \qquad \frac{10}{40} = \frac{1}{} \qquad \frac{36}{24} = 1\frac{}{2}$$

3. Complete the following without your calculator

$$\frac{1}{4} = \frac{}{8} \qquad \frac{2}{5} = \frac{}{20} \qquad \frac{3}{5} = \frac{}{25}$$

$$\frac{1}{3} = \frac{}{30} \qquad \frac{1}{2} = \frac{}{6} \qquad \frac{3}{4} = \frac{}{12}$$

4. Complete the following

$$\frac{6}{9} = \frac{}{3} \qquad \frac{25}{100} = \frac{5}{} \qquad \frac{}{4} = \frac{21}{28}$$

$$\frac{}{4} = \frac{10}{40} \qquad \frac{15}{25} = \frac{3}{} \qquad \frac{}{2} = \frac{36}{24}$$

5. In a group of 60 students, 27 were mature age students. What fraction of the group were mature age students? Can you re-express the fraction that you obtained with a smaller denominator?

Addition of Fractions

Using the strips take one $\frac{1}{6}$ and put it along side another $\frac{1}{6}$. Now find one single piece that is equal to these two pieces.

You will have found out that

$$\frac{1}{6} + \frac{1}{6} = \frac{1}{3} \qquad \text{Of course it is also equal to } \tfrac{2}{6}$$

Try the following:

$\frac{1}{12} + \frac{1}{12} =$	$\frac{1}{4} + \frac{1}{4} =$	$\frac{1}{12} + \frac{1}{6} + \frac{1}{12} =$
$\frac{1}{2} + \frac{1}{6} =$	$\frac{1}{3} + \frac{1}{6} =$	$\frac{1}{4} + \frac{1}{6} + \frac{1}{12} =$

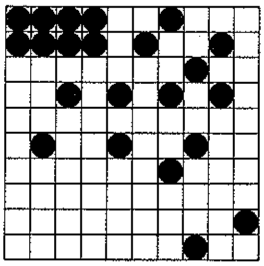
Multiplication of Fractions

1. What fraction of the whole will you have if you have 5 of the $\frac{1}{12}$ pieces?
2. What fraction of the whole will you have if you have 5 of the $\frac{1}{6}$ pieces?
3. What if you had 5 of the $\frac{1}{3}$ pieces?
4. How long will the resulting piece be if you fold the $\frac{1}{6}$ piece in half?
5. Express all of the above as multiplication of fractions.
6. Make up two examples of multiplication of fractions and give them to your neighbour to solve. Solve theirs.

Percentages

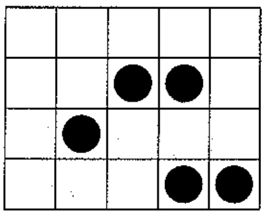
Percentages are fractions with a denominator of 100.

In the diagram below, what fraction of the total number of small squares have black blobs on them? i.e. what percentage of the squares have black blobs in them?

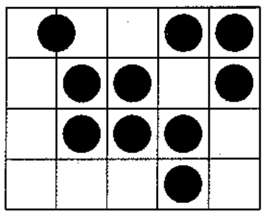


Often there will not be 100 squares or 100 people out of which to express a fraction or a percentage. When this is the case you will need to find an equivalent fraction out of 100 by multiplying by 100% or by multiplying by 1.

In the next figures, what percentage of the squares have circular shapes drawn in them?



Percentage circles =



Percentage circles =

As we have said previously a percentage is just a fraction with a denominator of 100, thus,

$$\frac{1}{20} = \frac{1 \times 5}{20 \times 5} = \frac{5}{100} = 5\% \quad \text{or} \quad \frac{1}{20} = \frac{1}{20} \times 100\% = \frac{100}{20}\% = 5\%$$

$$\frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100} = 30\% \quad \text{or} \quad \frac{3}{10} = \frac{3}{10} \times 100\% = \frac{300}{10}\% = 30\%$$

$$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 40\% \quad \text{or} \quad \frac{2}{5} = \frac{2}{5} \times 100\% = \frac{200}{5}\% = 40\%$$

Therefore you can either find out how many times the denominator goes into 100 and multiply the top and bottom of the fraction by this number **or** multiply the numerator by 100, then divide through by the denominator.

To convert each percentage to a fraction in its simplest form we can simplify the fraction that the percentage is equivalent to. Try these:

$80\% =$	$75\% =$	$37.5\% =$
$40\% =$	$85\% =$	$16\% =$
$35\% =$	$48\% =$	$62.5\% =$

9 Interchanging Fractions, Decimals and Percentages

Changing Fractions to Decimals

Using your calculator divide the top number (numerator) by the bottom number (denominator) to express a fraction as a decimal. If you do not have a calculator we can do it manually. Be sure to put a decimal point after the numerator and add a few zeros.

$$\frac{3}{4} \qquad \text{can be expressed as} \qquad 3.00 \div 4 = 0.75$$

Changing Fractions to Percentages

Express the fraction as a decimal (as shown above) then multiply the result by 100%. This can be done easily by shifting the decimal point two places to the right.

$$\frac{3}{8} = 3.000 \div 8 = 0.375 = 37.5\%$$

When the denominator of the fraction to be converted goes evenly into 100, the percentage can be found using equivalent fractions.

$$\frac{3}{5} = \frac{\quad}{100}$$

$$\frac{3}{5} \times \frac{20}{20} = \frac{60}{100} = 60\%$$

Converting Percentages to Decimals

In order to convert a percentage into a decimal, divide be 100. This can be done easily by shifting the decimal point two places to the left. If there is no decimal point be sure to place a decimal point to the right of the whole number.

$$45.8\% = 0.458 \qquad \text{and} \qquad 0.5\% = 0.005 \qquad \text{and} \qquad 7\% = 7.0\% = 0.07$$

Ordering Fractions, Decimals and Percentages

In order to compare numbers they must all be presented in the same form with the same number of decimal places. It is usually easiest to convert everything to decimals. Once you have done this, write the numbers underneath each other lining up the decimal points. Fill in any blanks with zeros. Compare the whole number side first. If there is a match, then compare the fractional side. For example: Express the following numbers in order from smallest to largest.

$$0.5,\ 5.3\%,\ 0.54,\ 47\%,\ \frac{3}{5},\ \frac{46}{100},\ 3$$

1. Convert all the numbers to decimals:

$$0.5, \quad 0.053, \quad 0.54, \quad 0.47, \quad 0.6, \quad 0.46, \quad 3$$

2. Line the numbers up

$$\begin{array}{l} 0.5 \\ 0.053 \\ 0.54 \\ 0.47 \\ 0.6 \\ 0.46 \\ 3 \end{array}$$

3. Fill in all of the blanks with zeros

$$\begin{array}{l} 0.5\mathbf{00} \\ 0.053 \\ 0.54\mathbf{0} \\ 0.47\mathbf{0} \\ 0.6\mathbf{00} \\ 0.46\mathbf{0} \\ 3.\mathbf{000} \end{array}$$

4. Compare the whole numbers. If there is a match, compare the fractional side. Since six of the numbers begin with a 0. we must compare the right hand side. Clearly

$$53 < 460 < 470 < 500 < 540 < 600$$

5. Express the numbers from smallest to largest

$$5.3\%, \quad \frac{46}{100}, \quad 47\%, \quad 0.5, \quad 0.54, \quad \frac{3}{5}, \quad 3$$

Multiplying Fractions

Before performing any operations on fractions always represent it in the form $\frac{a}{b}$ even if this makes it an improper fraction. Once all fractions are in this form simply multiply the numerators together and then multiply the denominators together. For example:

$$\text{One half of } 3\frac{3}{5} = \frac{1}{2} \times \frac{18}{5} = \frac{18}{10} = \frac{9}{5} = 1\frac{4}{5}$$

Finding a Percentage

When you are asked to find a percentage you must find the fraction first. For instance if there are 15 men, 13 women and 22 children in a group, the fraction of men in the group is $\frac{15}{50}$ since there are $50 = 15 + 13 + 22$ people in the group altogether. You can either multiply by 100 % (if you have a calculator) or find the equivalent fraction out of 100. In both cases you will reach an answer of 30%.

Ratio

Any fraction can also be expressed as a ratio. $\frac{15}{50}$ can be written as 15:50. However be careful of the wording of these types of questions. The ratio of men to the total number in the group is 15:50 but the ratio of men to women is 15:13.

Extension Activity

Can you write $\frac{35}{96}$ as a sum of fractions, each having numerator 1, and no two of them having the same denominator? How about $\frac{25}{42}$? Or $\frac{4}{25}$?

10 Introductory Algebra

In Mathematics we are continually looking for patterns and for relationships. In addition, we try to generalise and we can frequently do this with the help of algebra.
Try the examples below.

1. What are the next two numbers in the sequence?

   (a) 2,4,6,8,10,

   (b) 10,20,30,40,

   (c) 3,6,9,12,15,

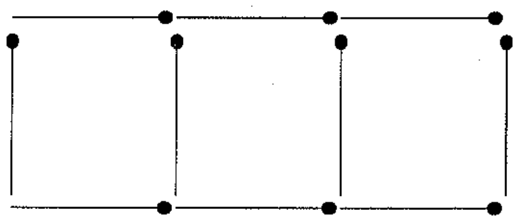
   (d) $10, 10^2, 10^3, 10^4$,

   (e) 1,4,9,16,25,36,

   (f) 1,2,4,7,11,16,22,

   (g) Make up a sequence of your own. Ask your neighbour if they can find the next two numbers in your sequence.

2. Use the matches to construct a string of three squares.



   How many matches did you use? Can you find some methodical way of counting the number of matches used? How many matches would you have used for

   (a) 5 squares

   (b) 8 squares

   (c) 16 squares

   (d) In words explain how the number of matches is related to the number of squares.

   How did your neighbour explain this? Did she use the same method as you? If she used a different method do you understand it? Find somebody else who used another method again. Hopefully you all arrived at the same answer even though you used different methods.

3. A Magic Square consists of numbers arranged in the form of a square so that the sum of the numbers in each row, in each column and along each diagonal is the same. Magic squares have been in existence for some thousands of years and were used as magic charms in Europe in the middle ages. The oldest magic square known is shown below.

4	9	2
3	5	7
8	1	6

Check that it is a magic square.
Now look at the square below, it has algebraic expressions in each of the smaller squares. By making the substitions fill in the lower square. (When we make a substitution we put in particular numbers in place of each of the letters. For example, substituting $x = 5$ and $y = 3$ into the expression $x + y$ gives $5 + 3$ which equals 8.)

$x - z$	$x + z - y$	$x + y$
$x + y + z$	x	$x - y - z$
$x - y$	$x + y - z$	$x + z$

Substitute $x = 5$, $y = 3$, $z = 1$ into the algebraic expressions above and put your results in the corresponding squares below.

What do you notice?

Try another set of values of x, y and z with $x > (y + z)$. What would happen if $x < (y + z)$? Try a set of values.

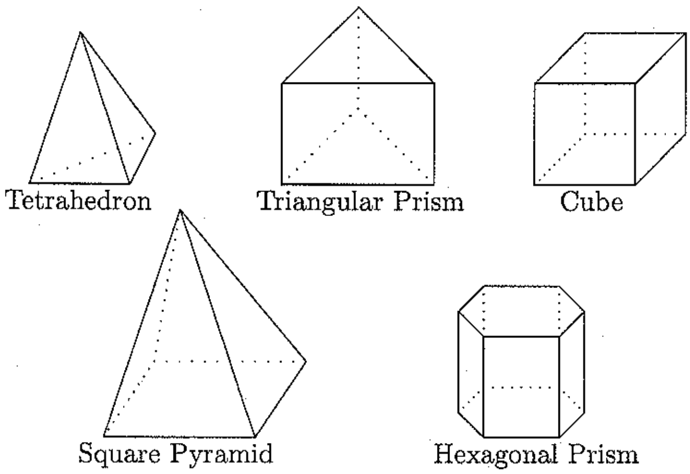
4. For each of the figures below

   (a) Count the number of vertices or corners (ie points)

   (b) Count the number of edges (knife edges)

   (c) Count the number of faces (flat surfaces)

   and use this information to complete the table below. Remember that there are vertices, edges and faces that you can't see.



Name of Figure	Number of vertices v	Number of edges e	Number of faces f	$f + v - e$

If you have counted correctly you should have found that $f + v - e$ is always equal to 2. You will therefore have verified **Euler's Theorem** which states that *for any solid polyhedron if we denote the number of faces by f, the number of vertices or corners by v and the number of edges by e. Then $f + v - e = 2$.*

11 Further Algebra

1. Take three cuisenaire rods all of the same colour. Let y be the length in cm of each rod. Put the rods in a line touching end to end. Use a ruler marked in centimetres to measure the length of the three rods. For your three rods, 3 lots of y equals what? You can express this as $3y =$

   In arithmetic we write 8×4. In algebra we could write $8 \times a$ where a is a variable but using our shorthand we instead often write $8a$. Instead of $1a$ we write a. Instead of $7 \times a$ we would write what?

   Compare your result for $3y$ with other members of your group. Probably they will have got different results to yourself. Why? We refer to y, which can take different values, as a variable.

2. Still using the same coloured rods make a model of $2y$. How long is it? Could you have worked this out without measuring it?

   By using the small centicubes model $2y+3$. Use a ruler measured in centimetres to find what this equals? What value did another member of your group get for this?

3. Model and draw $4y + 2$. What does it equal? Reaarange your model so that you have two equal halves. The value of course stays the same.

   Try to model and draw 2 lots of $2y + 1$. What does this equal? The mathematical way of writing this is $2(2y + 1)$. Compare this model with the one of $4y + 2$.

4. Complete the table below using different values for y. i.e. substitute y equals different values in each row of the table. The first one has been done for you.

   I had the black rod so my y equals 7.

y	$y+5$	$y+1$	$2(y+1)$	$2y+2$	$3y+6$	$3(y+2)$
7	12	8	16	16	27	27

5. $x+5$ is an algebraic expression and x is the variable. We can draw up a table of x-values and their corresponding $x+5$ values.

x	3	2	6	-1	-6			
$x+5$	8	7	11	-4	-1			

Think of some more values of x. Write them in the top line. Calculate the corresponding values of $x+5$ and write them in the corresponding place in the table.

Similarly we can draw up a table relating y and $3y^2+2$.

y	3	2	6	-1	-6			
$3y^2+2$	29	14	110	5	110			

Think of some more values of y. Write them in the top line. Calculate the corresponding values of $3y^2+2$ and write them in the corresponding place in the table.

Below we have a number of examples showing further the ability of algebra to generalise.

6. If z is an integer (i.e. a whole number) and we add 3 to it the new integer will be $z+3$.

   (a) What would the new integer be if we added 9 to it instead?

   (b) What would the new integer be if we had added 20 instead?

   (c) What would the new integer be if we had instead multiplied by 5?

7. In the following examples write an algebraic expression for each statement. The first is done for you.

   (a) Let p be a number, multiply it by 6 and then add 4 to the result.

$$6p+4$$

   (b) Let x be a number, then add 5

   (c) Let f be a number, then subtract 10.

   (d) Let q be a number, square it and then subtract 3.

   (e) Let y be a number, square it, multiply the result by 5 and then add 12.

8. Think of a number between one and ten. Double it. Add 8. Multiply by 3. Subtract 12. Divide by 6. Subtract the number your first thought of. The answer is?

   Try to model it with blocks. Now try it with algebra. What did you notice?

9. A three cube is a cube that has length 3, height 3 and width 3 i.e. it is a 3 by 3 by 3 cube. An n cube is a cube that had length n, height n and width n, i.e. it is an n by n by n cube. Working in groups and using the cubes provided, think about the following questions.

   (a) Suppose I construct a 3 cube. How many of the unit cubes would I need to use?

   (b) Suppose I now spray paint my 3 cube. How many of the small unit cubes would have three of their faces painted?

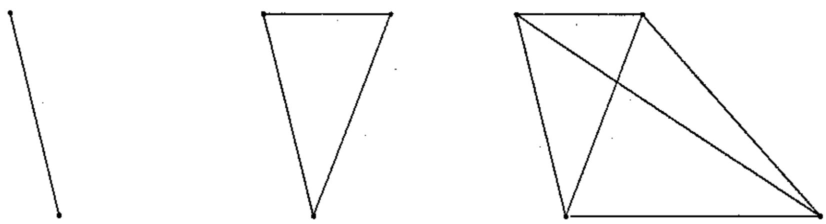
   (c) How many would have two of their faces painted?

   (d) How many would have one of their faces painted?

   (e) How many would have none of there faces painted?

   Answer the same questions for a four cube and a five cube. What would the answers be for a 10 cube? Can you come up with expressions to answer these questions for an n cube?

10. If we connect points, as shown in the pictures below, we need certain numbers of line segments, depending upon the number of points.



2 points - 1 line　　3 points - 3 lines　　4 points - 6 lines

   How many line segments would be needed (assuming the points are not collinear) to connect 5 points? 6 points? 10 points? 100 points? n points?