Test Four

This is a self-diagnostic test. Each question relates to a worksheet in a series available in the MUMS the WORD series. For example question 4 relates to worksheet 4.4 *Applications of Integration*. If you score 100% on this test and test 3 then we feel you are adequately prepared for your first year mathematics course. For those of you who had trouble with a few of the questions, we recommend working through the appproriate worksheets and associated computer aided learning packages in this series.

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1. (a) Differentiate $y = \log(3x + 2)$

   (b) Find $\frac{dy}{dx}$ if $y = x^2 \cos x$

2. (a) Given the following monotonically increasing function, find an upper and lower limit for the area under the curve between 0 and 4.

x	0	1	2	3	4
$g(x)$	2	3	5	6.5	7

   (b) Find the area under the curve $y = x^2 + 1$ between $x = 1$ and $x = 3$.

3. Evaluate the following indefinite integrals:

   (a) $\displaystyle\int \frac{1}{x}\,dx$

   (b) $\displaystyle\int \sec^2 x\,dx$

4. (a) Given $\dfrac{d^2x}{dt^2} = 9$ for all x and when $t = 0$ we have $\frac{dx}{dt} = 4$ and $x = 3$. What is x as a function of t?

   (b) A population $P(t)$ is given by the following formula:

$$P(t) = P(0)e^{kt}$$

   If the initial population is 1000, and the growth rate is 0.01, what is the population at $t = 100$? (You can leave the answer in terms of the natural exponential)

5. (a) What is the coefficient of x^2 in the expansion of $(5x-1)^5$?

   (b) Evaluate $\dfrac{6!}{4!2!}$.

   (c) How many 3-digit numbers can be formed from the digits 1,2,3,4,5,6, if repetition of digits are (i) allowed, (ii) not allowed.

6. (a) Write out the sum $\displaystyle\sum_{n=1}^{5} n^3$ without using sigma notation.

   (b) Write the sum $x^2 + 2x^4 + 3x^6 + ... + 10x^{20}$ in sigma notation.

7. Divide $6x^3 + x^2 - x + 4$ by $x + 1$.

8. (a) Simplify $\dfrac{\sin 4x}{(\cos^2 x - \sin^2 x)\sin x \cos x}$.

   (b) Find the exact value of $\cos \frac{\pi}{8}$.

9. Sketch $y = 2\sqrt{x-3} + 1$.

10. Let $f(x) = \dfrac{x+1}{x+2}$ and $g(x) = \sqrt{x}$.

    (a) Find $(f \circ g)(x)$.

    (b) Find $f^{-1}(x)$.

11. Let $f(x) = \dfrac{1}{e^x - 3}$.

    (a) Find the largest domain of f.

    (b) Find the inverse of f.

12. Use Mathematical Induction to prove that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

    for all $n \in \mathbb{N}$.

Answers to Test Four

1. (a) $\dfrac{3}{3x+2}$ (b) $2x\cos x - x^2\sin x$
2. (a) 16.5 and 21.5 (b) 32/3
3. (a) $\log x + C$ (b) $\tan x + C$
4. (a) $x = \frac{9}{2}t^2 + 4t + 3$ (b) $1000e$
5. (a) -250 (c) (i) 6^3 (ii) $6 \times 5 \times 4$
   (b) 15
6. (a) $1 + 2^3 + 3^3 + 4^3 + 5^3$ (b) $\displaystyle\sum_{n=1}^{10} nx^{2n}$
7. $6x^2 - 5x + 4$
8. (a) 4
   (b) $\sqrt{\dfrac{1+\sqrt{2}}{2\sqrt{2}}}$
10. (a) $\dfrac{\sqrt{x}+1}{\sqrt{x}+2}$ (b) $\dfrac{1}{1-x} - 2$
11. (a) $x \neq \log_e 3$ (b) $\log_e\left(\frac{1}{x} + 3\right)$