Test Four

This is a self-diagnostic test. Each question relates to a worksheet in a series available in the MUMS the WORD series. For example question 4 relates to worksheet 4.4 Applications of Integration. If you score 100% on this test and test 3 then we feel you are adequately prepared for your first year mathematics course. For those of you who had trouble with a few of the questions, we recommend working through the appropriate worksheets and associated computer aided learning packages in this series.

1. (a) Differentiate \( y = \log(3x + 2) \)
   (b) Find \( \frac{dy}{dx} \) if \( y = x^2 \cos x \)

2. (a) Given the following monotonically increasing function, find an upper and lower limit for the area under the curve between 0 and 4.

   \[
   \begin{array}{c|c|c|c|c|c}
   x & 0 & 1 & 2 & 3 & 4 \\
   g(x) & 2 & 3 & 5 & 6.5 & 7 \\
   \end{array}
   \]

   (b) Find the area under the curve \( y = x^2 + 1 \) between \( x = 1 \) and \( x = 3 \).

3. Evaluate the following indefinite integrals:
   (a) \( \int \frac{1}{x} \, dx \)
   (b) \( \int \sec^2 x \, dx \)

4. (a) Given \( \frac{d^2x}{dt^2} = 9 \) for all \( x \) and when \( t = 0 \) we have \( \frac{dx}{dt} = 4 \) and \( x = 3 \). What is \( x \) as a function of \( t \)?
   (b) A population \( P(t) \) is given by the following formula:
   \[
P(t) = P(0)e^{kt}
   \]
   If the initial population is 1000, and the growth rate is 0.01, what is the population at \( t = 100 \)? (You can leave the answer in terms of the natural exponential)
5. (a) What is the coefficient of \(x^2\) in the expansion of \((5x - 1)^5\)?

(b) Evaluate \(\frac{6!}{4!2!}\).

(c) How many 3-digit numbers can be formed from the digits 1,2,3,4,5,6, if repetition of digits are (i) allowed, (ii) not allowed.

6. (a) Write out the sum \(\sum_{n=1}^{5} n^3\) without using sigma notation.

(b) Write the sum \(x^2 + 2x^4 + 3x^6 + \ldots + 10x^{20}\) in sigma notation.

7. Divide \(6x^3 + x^2 - x + 4\) by \(x + 1\).

8. (a) Simplify \(\frac{\sin 4x}{(\cos^2 x - \sin^2 x) \sin x \cos x}\).

(b) Find the exact value of \(\cos \frac{\pi}{8}\).

9. Sketch \(y = 2\sqrt{x - 3} + 1\).

10. Let \(f(x) = \frac{x + 1}{x + 2}\) and \(g(x) = \sqrt{x}\).

(a) Find \((f \circ g)(x)\).

(b) Find \(f^{-1}(x)\).

11. Let \(f(x) = \frac{1}{e^x - 3}\).

(a) Find the largest domain of \(f\).

(b) Find the inverse of \(f\).

12. Use Mathematical Induction to prove that

\[1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)\]

for all \(n \in \mathbb{N}\).
Answers to Test Four

1. (a) \( \frac{3}{3x + 2} \) \hspace{1cm} (b) \( 2x \cos x - x^2 \sin x \)

2. (a) 16.5 and 21.5 \hspace{1cm} (b) \( \frac{32}{3} \)

3. (a) \( \log x + C \) \hspace{1cm} (b) \( \tan x + C \)

4. (a) \( x = \frac{9}{2}t^2 + 4t + 3 \) \hspace{1cm} (b) \( 1000e \)

5. (a) -250 \hspace{1cm} (b) 15 \hspace{1cm} (c) (i) \( 6^3 \) (ii) \( 6 \times 5 \times 4 \)

6. (a) \( 1 + 2^3 + 3^3 + 4^3 + 5^3 \) \hspace{1cm} (b) \( \sum_{n=1}^{10} nx^{2n} \)

7. \( 6x^2 - 5x + 4 \)

8. (a) 4 \hspace{1cm} (b) \( \sqrt{\frac{1 + \sqrt{2}}{2\sqrt{2}}} \)

10. (a) \( \frac{\sqrt{x} + 1}{\sqrt{x} + 2} \) \hspace{1cm} (b) \( \frac{1}{1 - x} - 2 \)

11. (a) \( x \neq \log_e 3 \) \hspace{1cm} (b) \( \log_e (\frac{1}{x} + 3) \)