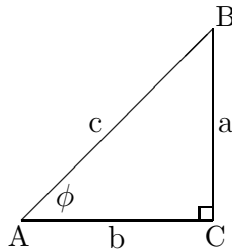


Worksheet 3.3 Trigonometry

Section 1 REVIEW OF TRIG RATIOS

Worksheet 2.8 introduces the trig ratios of sine, cosine, and tangent. To review the ratios, consider a triangle ABC with angle ϕ as marked.



The hypotenuse (hyp) of the triangle is c ; the adjacent (adj) side is b ; the opposite (opp) side is a . The side of length a is opposite the angle, and the side of length b is the side adjacent to the angle which is *not* the hypotenuse. Then we have

$$\begin{aligned}\sin \phi &= \frac{\text{opp}}{\text{hyp}} = \frac{a}{c} \\ \cos \phi &= \frac{\text{adj}}{\text{hyp}} = \frac{b}{c} \\ \tan \phi &= \frac{\text{opp}}{\text{adj}} = \frac{a}{b}\end{aligned}$$

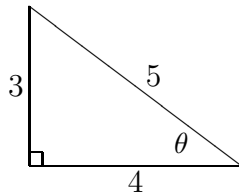
Note also that

$$\frac{\sin \phi}{\cos \phi} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{b} = \tan \phi$$

Exercises:

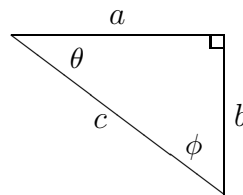
1. For the following triangle, find the ratios:

- (a) $\sin \theta$
- (b) $\tan \theta$
- (c) $\cos \theta$

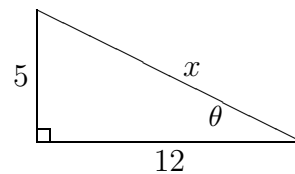


2. For the following triangle, find the ratios:

- | | | | |
|-----|---------------|-----|---------------|
| (a) | $\tan \theta$ | (d) | $\sin \phi$ |
| (b) | $\cos \phi$ | (e) | $\tan \phi$ |
| (c) | $\sin \theta$ | (f) | $\cos \theta$ |



3. (a) Use Pythagoras' theorem to find x
 (b) Find (i) $\sin \theta$
 (ii) $\tan \theta$
 (iii) $\cos \theta$



Section 2 DEGREES AND RADIANS

Recall from Worksheet 2.9 that

$$1^\circ = \frac{\pi}{180} \text{ radians}$$

In university maths it is much more common to give angles in radians rather than degrees. If the units are left off an angle, then the angle is in radians. Degrees can be converted to radians using the above formula, but it will be very convenient for you to know some standard conversions. In particular:

$$\begin{array}{ll} 90^\circ = \frac{\pi}{2} & 30^\circ = \frac{\pi}{6} \\ 45^\circ = \frac{\pi}{4} & 180^\circ = \pi \\ 60^\circ = \frac{\pi}{3} & 360^\circ = 2\pi \end{array}$$

Example 1 : An equilateral \triangle has three equal angles of $\frac{\pi}{3}$.

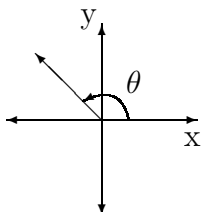
Example 2 : Convert 50° to radians.

$$50^\circ = 50 \frac{\pi}{180} = \frac{5\pi}{18} \text{ radians}$$

Example 3 : How many degrees is $\frac{\pi}{9}$ radians? We know $180^\circ = \pi$, so $\frac{\pi}{9} = \frac{180^\circ}{9}$ therefore

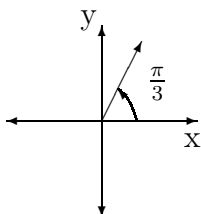
$$\frac{\pi}{9} = 20^\circ$$

If we think of an angle θ as the amount of rotation of a straight line about the angle, then we can define a positive rotation as one which is anti-clockwise and a negative rotation as one which is in a clockwise direction. We can see the ordinary $x - y$ plane with the vertex of the angle at the origin and the base of the angle beginning at the positive x -axis. i.e. the positive x -axis represents the position of the line if the angle of rotation is 0. So for the angle θ we get the following diagram:

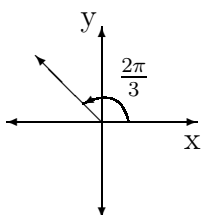


We can now represent angles graphically and we can deal with angles of any size. Recall that a full revolution is 2π , or 360° . So rotating a line through 2π will bring it back to its starting position.

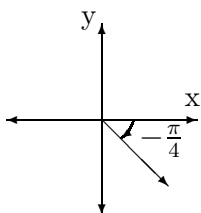
Example 4 : Represent $\frac{\pi}{3}$ radians graphically. Since π is the angle half way round the plane, $\frac{\pi}{3}$ is the angle one third of the way around the upper half of the plane.



Example 5 : Represent $\frac{2\pi}{3}$ radians graphically.



Example 6 : Represent $-\frac{\pi}{4}$ radians graphically.



Exercises:

1. Write the following degrees in radian measure

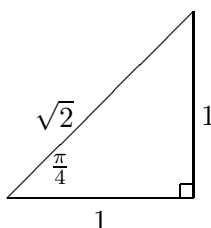
- (a) 80° (b) 120° (c) 90° (d) 270° (e) 135°

2. Convert the following radian measures to degrees

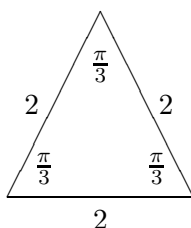
- (a) $\frac{\pi}{8}$ (b) $\frac{2\pi}{9}$ (c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{6}$ (e) $\frac{7\pi}{6}$

Section 3 STANDARD TRIANGLES

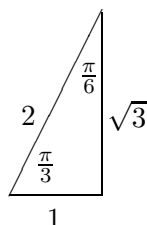
There are two triangles which are known as standard triangles. These triangles and the information contained in them should be memorized, as you will be expected to know certain information without using a calculator. The first triangle is a right-angled isosceles triangle. Recall that an isosceles triangle has two angles the same and two sides the same length:



The associated trig ratios are fairly simple to work out, and are left as exercises. The second standard triangle is half an equilateral triangle of length 2.



When we have taken half the equilateral triangle, we end up with the following:



Pythagoras' theorem gives us the length of the vertical side as $\sqrt{3}$, and the angle ϕ is half the top angle so $\phi = \frac{\pi}{6}$. The trig ratios given by this triangle are:

$$\begin{array}{ll} \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} & \sin \frac{\pi}{6} = \frac{1}{2} \\ \cos \frac{\pi}{3} = \frac{1}{2} & \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \\ \tan \frac{\pi}{3} = \sqrt{3} & \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \end{array}$$

Once the triangles are memorized the trig ratios can be found by just drawing either of the two triangles. It is important that you memorize the trig ratios for the angles $\frac{\pi}{6}$, $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

Exercises:

1. Find the exact ratios for the following

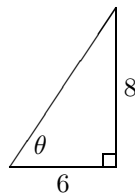
$$(a) \tan \frac{\pi}{3} \quad (b) \cos \frac{\pi}{6} \quad (c) \sin \frac{\pi}{4} \quad (d) \tan \frac{\pi}{4} \quad (e) \tan \frac{\pi}{6}$$

2. Use exact ratios to find θ in each of the following equations, where $0 \leq \theta \leq \frac{\pi}{2}$.

$$\begin{array}{lll} (a) \sin \theta = \frac{1}{2} & (c) \tan \theta = 1 & (e) \cos \theta = \frac{1}{\sqrt{2}} \\ (b) \cos \theta = \frac{1}{2} & (d) \sin \theta = \frac{\sqrt{3}}{2} & \end{array}$$

Section 4 USING TRIGONOMETRIC RATIOS

We can use the trigonometric ratios described in the previous sections to find an unknown angle or side in a right-angled triangle. Consider the following triangle:

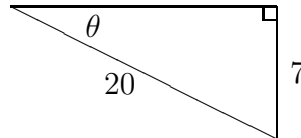


Let us say that we wish to find θ in this triangle. The side that is 6 units long is adjacent to θ ; the side that is 8 units long is opposite to θ , so we have

$$\begin{aligned} \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{8}{6} \\ &= \frac{4}{3} \end{aligned}$$

Consequently, θ is an angle whose tangent is $\frac{4}{3}$. That is $\theta = \tan^{-1} \frac{4}{3}$. By using the \tan^{-1} button on a calculator, we find that $\theta = 0.927$, to three decimal places. Note that this answer is in radians.

Example 1 : What is θ ?

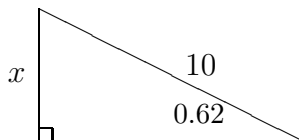


With respect to θ , the opposite side is 7 units long, and the hypotenuse is 20 units long. Therefore,

$$\begin{aligned} \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{7}{20} \\ \theta &\doteq 0.358 \end{aligned}$$

The last step was carried out using the \sin^{-1} button on a calculator, and the answer is approximate and in radians.

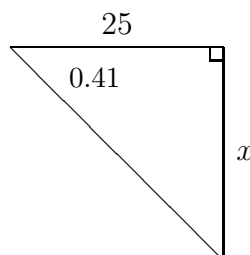
Example 2 : What is x ?



The trigonometric ratios may also be used to find the length of a side in a right angled triangle.

$$\begin{aligned} \sin 0.62 &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{x}{10} \\ x &= 10 \sin 0.62 \\ x &= 5.81 \end{aligned}$$

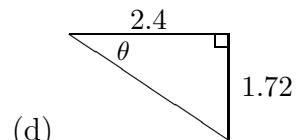
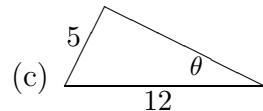
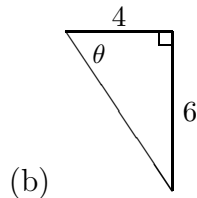
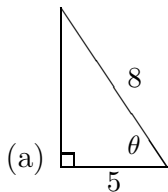
Example 3 : What is x ?



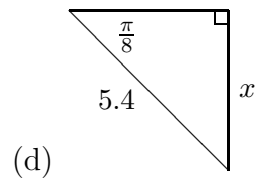
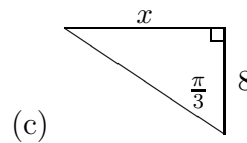
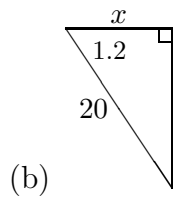
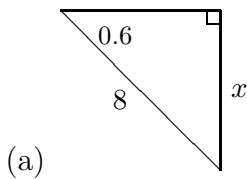
$$\begin{aligned}
 \tan 0.41 &= \frac{\text{OPP}}{\text{ADJ}} \\
 &= \frac{x}{25} \\
 x &= 25 \tan 0.41 \\
 x &= 10.87
 \end{aligned}$$

Exercises:

1. Find θ in each of the following



2. Find x , to 2 decimal places, in each of the following triangles.



Section 5 INVERSE TRIG FUNCTIONS

Sometimes you will come across the notation $\sin^{-1} a$ or $\cos^{-1} a$. Now, $\sin^{-1} a$ does not mean $\frac{1}{\sin a}$. It is called the arcsine of a , and means this: the sin of what angle will give an answer a ? So

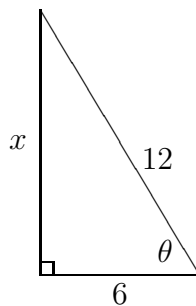
$$\begin{aligned}
 \sin^{-1} a &= \theta \text{ means} \\
 \sin \theta &= a
 \end{aligned}$$

The same rule applies to $\cos^{-1} a$ and $\tan^{-1} a$. If you wish to write $\frac{1}{\sin a}$ then you would do so as $(\sin a)^{-1}$ so that there is no confusion.

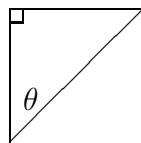
Exercise 3.3 Trigonometry

1. (a) Covert
 - i. $\frac{5\pi}{6}$ to degrees
 - ii. $\frac{12\pi}{9}$ to degrees
 - iii. 80° to radians; write the answer as a number times π .
 - iv. 42° to radians; write the answer to 3 decimal places
- (b) Find the exact values of
 - i. $\sin \frac{\pi}{4}$
 - ii. $\cos \frac{\pi}{6}$
 - iii. $\tan \frac{\pi}{3}$

2.



- (a) Find the value of x
 - (b) Evaluate θ
3. Joan walks 5km north, then 3.6km east.
- (a) Put these distances onto the appropriate sides of the triangle below:



- (b) Find the angle θ , the bearing that Joan has effectively walked along.

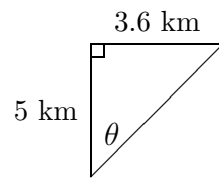
Answers 3.3

1. (a) i. 150° ii. 240° iii. $\frac{4\pi}{9}$ iv. $\frac{7\pi}{30} = 0.733$

(b) i. $\frac{1}{\sqrt{2}}$ ii. $\frac{\sqrt{3}}{2}$ iii. $\sqrt{3}$

2. (a) $\sqrt{108}$ (b) $\frac{\pi}{3}$

3. (a)



(b) 35.75°