Test Three

This is a self-diagnostic test. Every pair of questions relates to a worksheet in a series available in the MUMS the WORD series. For example question 5 relates to worksheet 3.5 Simultaneous Equations. If you score 100% on this test and test 4 then we feel you are adequately prepared for your first year mathematics course. For those of you who had trouble with a few of the questions, we recommend working through the appropriate worksheets and associated computer aided learning packages in this series.

1. (a) If \( f(x) = x^2 + 2 \), what is \( f(3x) \)?
   (b) If \( g(x) = x^2 \), and \( f(x) = 2x + 1 \), what is \( g \circ f(x) \)?

2. (a) What is the domain of \( g(x) \) in the following graph?

   \[ g(x) \]

   \[ -4 \quad -2 \quad 2 \quad 4 \]

   (b) At what points on the graph above does \( g(x) = 0 \)?

3. (a) If an angle is 60°, how many radians is it?
   (b) For the \( \triangle \) drawn below, what is the angle \( \phi \)?

   \[ \triangle \]

   \[ 2 \quad \phi \quad \sqrt{3} \]

4. (a) What is \( \sin \frac{\pi}{4} \)? (Without a calculator)
   (b) Find the area of the triangle drawn below.

   \[ \triangle \]

   \[ 2 \quad \sqrt{3} \quad 2 \]
5. (a) Given:

\[ y = 2x + 5 \]
\[ y = kx + 4 \]

For what value(s) of \( k \) will the system have no solutions?

(b) Solve the system:

\[ y = 3u + 6 \]
\[ 3y = 5u + 2 \]

6. (a) If 2, 7 are the first two terms of an arithmetic progression, what is the 10th term?

(b) What is the sum of the following infinite geometric series?

\[ 1 + \frac{9}{10} + \frac{81}{100} + \ldots \]

7. (a) Find the limit of

\[ \lim_{n \to 5} \frac{x^2 - 25}{x - 5} \]

(b) Is the function

\[ f(x) = \begin{cases} 
\frac{x^2 + 6}{x} & x \neq 0 \\
0 & x = 0 
\end{cases} \]

continuous at \( x = 0 \)? Why?

8. (a) Find the derivative of \( f(x) = x^3 + 3x^2 + 3 \).

(b) What are the stationary points of the function \( g(x) = x^3 + 3x^2 \)?

9. (a) What sort of turning points does the function \( f(x) = 6 - 3x^2 \) have?

(b) When does the concavity change for the function \( h(x) = x^3 + x^2 + 5x + 2 \)?

10. (a) Differentiate \( y = \sin(5x + 3) \).

(b) Differentiate \( y = 3e^x \).
Answers to Test Three

1. (a) $9x^2 + 2$
   (b) $(2x + 1)^2$

2. (a) $[-4, 0] \cup (2, 4]$
   (b) $x = -4$

3. (a) $\frac{\pi}{3}$
   (b) $\phi = \frac{\pi}{6}$

4. (a) $-\frac{1}{\sqrt{2}}$
   (b) $\sqrt{3}$

5. (a) $k = 2$
   (b) $u = -4, y = -6$

6. (a) 47
   (b) 10

7. (a) 10
   (b) No

8. (a) $3x^2 + 6x$
   (b) $(0, 0), (-2, 4)$

9. (a) Max
   (b) $x = -\frac{1}{3}$

10. (a) $5\cos(5x + 2)$
    (b) $3e^x$