Worksheet 1.9 Introduction to Algebra

Section 1 Algebraic Expressions

Algebra is a way of writing arithmetic in a general form. You have already come across some algebraic expressions in previous worksheets. An algebraic expression is one in which the arithmetic is written with symbols rather than numbers. The most common use of algebra is in writing formulae. A formula is an algebraic expression which acts as a general 'recipe':

Example 1 :

$$C = 2.54 \times I$$

where I represents the number of inches, and C represents the number of centimetres. This formula represents a recipe for converting a length in inches to one in centimetres. If a ruler, say, is 12 inches long, then we can use the formula to work out how long it is in centimetres. It is $2.54 \times 12 = 30.48$cm.

Example 2 : You want to buy tickets for a show for 2 adults and 2 children. Let the price for an adult ticket be a (in dollars) and the price for a child c (again in dollars). Then the total cost P is

$$P = a + a + c + c$$

If the adult's tickets are \$55, and the children's tickets are \$30, then we would have

$$\begin{aligned} a &= 55 \\ c &= 30 \end{aligned}$$

so that $P = 55 + 55 + 30 + 30$. But if you realize that $P = 2a + 2b = 2(a + b)$ then you have an easier calculation to do, and it is also easy to substitute in other prices for the tickets.

Here are some algebraic expressions that we have already seen in the worksheets:

1. ab means a multiplied by b
2. $(-a)b = -ab$ means $-a$ multiplied by b
3. $2(x + y)$ means the sum of x and y all multiplied by 2

4. $2x + y$ means y added to 2 lots of x
5. $xx = x^2$ means x multiplied by itself
6. $\frac{p}{q}$ and p/q both mean p divided by q
7. $\frac{a}{x+y}$ means a divided by the sum of x and y

Note that multiplication signs are often omitted or replaced by brackets. When we multiply two numbers, say 3 and 5, we must write 3×5 or $(3)5$ rather than 35, which is indistinguishable from the number thirty-five. Remember that, when we are dealing with numbers, symbols represent numbers. This means that $a \times b = b \times a$ and that $(ab)c = a(bc) = abc$.

Section 2 Simplifying Algebraic Expressions

In many algebraic expressions we look for ways of simplifying, or tidying up, the expression so that it appears in its most compact form. In our previous example about the price of tickets,

$$P = 2(a + c)$$

is much neater than

$$P = a + a + c + c$$

The first step in many such simplifications is to collect like terms. The terms in an algebraic expression are the parts that are separated by + and − signs. For instance, in the expression

$$5a + 3c + 2d - 7a$$

the terms are $5a$, $3c$, $2d$ and $7a$. The terms which have exactly the same letters in them are called *like terms*.

Example 1 :
In the expression

$$7xy - 3x + 2xy + 4x - 5y$$

$7xy$ and $2xy$ are like terms and $-3x$ and $4x$ are also like terms. Our expression can be simplified as follows:

$$\begin{aligned} 7xy - 3x + 2xy + 4x - 5y &= 7xy + 2xy + 4x - 3x - 5y \\ &= 9xy + x - 5y \end{aligned}$$

Example 2 :
In the expression

$$5x^2 - 2x + 7x^2$$

the terms are $5x^2$, $2x$, and $7x^2$; the like terms are $5x^2$ and $7x^2$. The whole expression can be simplified:

$$\begin{aligned} 5x^2 - 2x + 7x^2 &= 5x^2 + 7x^2 - 2x \\ &= 12x^2 - 2x \end{aligned}$$

Collecting like terms means to bring them together as a single term; an example of this was replacing $a + a$ with $2a$ with the ticket example. Then $3x^3 + 5x^3$ can be replaced with $8x^3$. Notice how the sign in front of the term remains with the term, and, where there is no sign, a positive term is implied. Then $2x - 6x = -4x$.

Example 3 :

$$\begin{aligned} 5x^2 + 3x + 2x^2 - x &= 5x^2 + 2x^2 + 3x - x \\ &= 7x^2 + 2x \end{aligned}$$

Example 4 :

$$\begin{aligned} \frac{1}{3}x + \frac{1}{4}x &= (\frac{1}{3} + \frac{1}{4})x \\ &= (\frac{4}{12} + \frac{3}{12})x \\ &= \frac{7}{12}x \end{aligned}$$

Exercises:

1. Simplify
   (a) $3x - 2x + 4x$
   (b) $-xy + 2xy + x$
   (c) $2x^2y + x^2 - y^2 + 3y^2$
   (d) $\frac{1}{2} - x^2 - \frac{1}{3}x + x^2$
   (e) $5 + x - 3 + y + 6x + 2$
   (f) $xyz + yz + xz - 3yz$

Section 3 Removal of Brackets

Whatever is inside brackets should be treated as a single term. If you have an expression which involves only addition and subtraction there are two rules to remember.

1. An addition sign, or plus sign, in front of the brackets leaves the sign of every term inside the brackets unchanged.
2. A subtraction sign in front of the bracket indicates that, when removing the bracket, the sign of all terms inside must be changed.

Then

$$\begin{aligned} -(a+b) &= -a-b \\ -(a-b) &= -a+b \\ -(-a-b) &= a+b \\ (-a+b) &= -a+b \end{aligned}$$

If there is no sign before the brackets, a positive sign is implied as is the case with other terms. The removal of brackets is called expanding.

Example 1 :

$$\begin{aligned} -(x+2)+x &= -x-2+x \\ &= = -x+x-2 \\ &= -2 \end{aligned}$$

Often an algebraic expression will be simplified by expanding the bracketed terms and collecting terms.

Example 2 :

$$\begin{aligned} 3+5x-(2x+3)+5y &= 3+5x-2x-3+5y \\ &= 3x+5y \end{aligned}$$

Exercises:

1. Simplify
   (a) $-(x+2)+(x-1)$
   (b) $x^2+x-(x^2-x)$
   (c) $\frac{1}{2}(x+y)-\frac{1}{2}(x-y)$
   (d) $2x^2+y^2-\frac{1}{4}(x^2+y^2)$
2. Simplify
   (a) $(x+1)^2+(x-1)^2$
   (b) $(2x+y)^2-(2x+y)$
   (c) $x^2-(x+y)^2$
   (d) $\frac{1}{4}x^2+(\frac{x+1}{2})^2$

Exercises 1.9 Introduction to Algebra

1. Simplify the following:

   (a) $2a + 5a$

   (b) $6a + 2b - 3a$

   (c) $6x - 7x + 8x^2$

   (d) $-(x + y) + 2x$

   (e) $-(x - y) + y$

   (f) $4ab + 6a - 2ab - 4a$

   (g) $3x + (x - y) + y$

   (h) $2x^3 - 3x^2 + 2x^2 - 7$

   (i) $(-p + q) - (-p - q)$

   (j) $\frac{1}{3}x + \frac{1}{4}y - \frac{2}{3}x - \frac{3}{8}y$

2. (a) Jason wants to buy p books at \$15 each. Write a mathematical sentence to find the total cost C of the books.

   (b) I intend buying b avocados at 90 cents each and r rockmelons at \$1.20 each. Write a mathematical sentence to find the total cost C of the purchase.

   (c) Alex has m children and Yvonne has n children. Write a mathematical sentence to find the total number of children, N, that they have between them.

   (d) Greg lives 4 times as far from the university as Jerry. If Jerry lives w kilometres away, write an expression to show the number of kilometres, t, that Greg lives from the university.

Answers 1.9

Section 2

1. (a) $5x$ (b) $xy + x$ (c) $2x^2y + x^2 + 2y^2$ (d) $\frac{1}{2} - \frac{1}{3}x$ (e) $4 + 7x + y$ (f) $xyz - 2yz + xz$

Section 3

1. (a) -3 (b) $2x$ (c) y (d) $\frac{7}{4}x^2 - \frac{3}{4}y^2$

2. (a) $2x^2 + 2$ (b) $(2x + y)(2x + y - 1)$ (c) $-2xy - y^2$ (d) $\frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{4}$

Exercises 1.9

1. (a) $7a$ (b) $3a + 2b$ (c) $8x^2 - x$ (d) $x - y$ (e) $2y - x$ (f) $2ab + 2a$ (g) $4x$ (h) $2x^3 - x^2 - 7$ (i) $2q$ (j) $-\frac{1}{3}x - \frac{1}{8}y$

2. (a) $C = 15p$ (b) $C = 0.9b + 1.2r$ (c) $N = n + m$ (d) $t = 4w$