Worksheet 1.5 Percentages

Section 1 Proportions

A pie is cut into twelve pieces. John eats five pieces, Peter eats one piece and Chris and Michael eat three pieces each. If we ask what proportion of the pie John ate we are asking what fraction of the total pie he ate. The proportion John ate is $\frac{5}{12}$.

Proportions are comparisons, usually between part of something and the whole of it. Chris and Michael each had $\frac{3}{12}$ or $\frac{1}{4}$ of the pie.

For the proportion of pie John ate you could instead say that he ate 5 in 12 parts of the pie. This means for every twelve pieces of pie John ate five. To write it this way instead of the fractional way we write 5:12 and say five in twelve. Proportions can be simplified in the same way as fractions, i.e. by canceling common factors.

Example 1 : For every hundred students enrolled in first-year Maths at a university 55 of them are males. What proportion of first-year Maths students at the university are female?

$$\begin{aligned} 100 - 55 &= 45 \text{ females in every } 100 \\ \text{proportion of females} &= \frac{45}{100} \\ \frac{45}{100} &= \frac{9\times \cancel{5}}{20\times \cancel{5}} = \frac{9}{20} \end{aligned}$$

So the proportion of females in first-year Maths is $\frac{9}{20}$ or 9:20. Hence we could say that for every 20 students enrolled in the first year maths course, 9 of them are female.

Example 2 : In a week a car dealer sells 10 red cars, 8 blue cars, 20 white cars and 2 black cars. What proportion of cars sold were red? What proportion were not black? Well, $10 + 8 + 20 + 2 = 40$ cars were sold in a week. But 10 cars were red so the proportion of red cars sold is

$$\frac{10}{40} = \frac{1}{4}$$

which can also be denoted 1:4. The proportion of cars sold that were not black is

$$\frac{40-2}{40} = \frac{38}{40} = \frac{19}{20} \quad \text{or} \quad 19:20.$$

Exercises:

1. In a class of 28 students, 16 were boys.
   (a) What proportion of the students were boys?
   (b) What proportion of the class were girls?
2. A group of 50 people were interviewed, who worked in the CBD of Edge city. Of the 50: 24 traveled by bus to work, 8 by car, and 18 by train. What proportion
   (a) Traveled by bus
   (b) Did not travel by bus
   (c) Traveled by car
   (d) Traveled by bus or train

Section 2 Percentages and Decimals

Percentages are another way of talking about proportions. When comparing proportions you may end up with long lists of varying denominators, so it is simpler to standardise the denominator for comparing proportions. The standard denominator is 100 and a percentage is a number out of 100 (per cent meaning out of 100 in Latin). Thus 80% means a proportion of 80 in 100 or $\frac{80}{100}$.

To convert proportions to percentages, then, is a matter of finding an equivalent fraction with denominator 100. To review equivalent fractions see worksheet 1.3. The percentage is the numerator of a fraction which has denominator 100.

Example 1 : Express $\frac{4}{5}$ as a percentage.

$$\frac{4}{5} = \frac{4}{5} \times \frac{20}{20} = \frac{80}{100} = 80\%$$

Notice that we get the same answer if we do the calculation this way:

$$\frac{4}{5} = \frac{4 \times 100}{5} \times \frac{1}{100} = \frac{400}{5} \times \frac{1}{100} = 80 \times \frac{1}{100} = 80\%$$

So if we wish to convert a proportion to a percentage we can simply multiply by $\frac{100}{1}$ to get a percentage amount.

Example 2 : Express $\frac{3}{4}$ as a percentage.

$$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 75\%$$

Express $\frac{1}{8}$ as a percentage.

$$\frac{1}{8} \times \frac{100}{1} = \frac{100}{8} = 12.5\%$$

Since a percentage is the numerator of a fraction with a denominator of 100 they can also be expressed as decimals.

Example 3 :

$$\begin{aligned} 0.07 &= \frac{7}{100} = 7\% \\ 0.12 &= \frac{12}{100} = 12\% \\ 217\% &= \frac{217}{100} = 2.17 \end{aligned}$$

Exercises:

1. Convert the following to percentages:

   (a) $\frac{9}{10}$
   (b) $\frac{20}{100}$
   (c) $\frac{3}{8}$
   (d) $\frac{3}{5}$
   (e) $\frac{17}{20}$

2. Convert the following percentages to fractions, and simplify where necessary:

   (a) 24%
   (b) 60%
   (c) 45%
   (d) 15%
   (e) $8\frac{1}{2}\%$

3. Convert the following percentages to decimals

   (a) 64%
   (b) 8%
   (c) 21.5%
   (d) 19%
   (e) 2.4%

Section 3 Problems relating to Percentages

Often you will be asked to find a particular percentage of a quantity. For example you might need to find 20% of \$500. To do this you use multiplication of fractions. The percentage is expressed in its equivalent fraction form and you then multiply it by the quantity to get the answer.

Example 1 : Find 20% of \$500

$$\frac{20}{100} \times \frac{500}{1} = \frac{20 \times 5 \times 100}{100 \times 1} = 100$$

So 20% of \$500 is \$100.

Example 2 : A jacket in a shop costs \$60. It is marked down by 5%. How much will you pay for the jacket?

First find 5% of \$60 and then subtract the answer from the price of the jacket. Alternatively find 95% of \$60. This is how much you will pay for the jacket.

$$\frac{5}{100} \times \frac{60}{1} = \frac{300}{100} = 3$$

So 5% of \$60 is \$3. The jacket will sell for \$60-\$3=\$57.

Alternatively

$$\frac{95}{100} \times \frac{60}{1} = \frac{95 \times 60}{100} = 19 \times 3 = 57$$

The jacket will sell for \$57. That is, the jacket will sell for 95% of the original price.

You can choose either of the methods illustrated above to get the answer.

Example 3 : The price of a clock which costs \$80 is to be increased by 15%.

Method A: Find 15% of \$80 and add this amount to \$80.

$$\frac{15}{100} \times \frac{80}{1} = \frac{15}{5} \times \frac{4}{1} = \frac{60}{5} = 12$$

Hence %15 of \$80 is \$12. The price of the clock is increased by \$12 to \$92.

Method B: Find %115 of \$80. (%115 = %100 + %15).

$$\frac{115}{100} \times \frac{80}{1} = \frac{115}{5} \times \frac{4}{1} = \frac{460}{5} = 92$$

So the price of the clock is increased to \$92.

Some questions might give you a percentage with a corresponding amount and ask you to work out what the total quantity is. If you take the quantity given and divide by the percentage you get the quantity equivalent to 1%. Now multiply by 100 and you will have the amount corresponding to 100%.

Example 4 : A car is marked down to 75% of its original price. It now costs \$15000. What was its original price?

That is,

$$\begin{aligned} 75\%\text{of the original price} &= 15000 \\ 1\%\text{of the original price} &= \frac{\text{Quantity}}{\text{Percentage}} \\ &= \frac{15000}{75} \\ &= \frac{150 \times 100}{75} \\ &= \frac{2 \times 100}{1} \\ &= 200 \end{aligned}$$

So \$200 is 1% of the original price. That is,

$$\begin{aligned} 1\%\text{of the original price} &= 200 \\ 100\%\text{of the original price} &= 200 \times 100 \\ &= 20000 \end{aligned}$$

Therefore the original price of the car was \$20000.

Exercises:

1. Find the following percentages:
   (a) 40% of 700 ml
   (b) 65% of $8
   (c) 32% of 6L (put your answer in mL)
2. Perform the following changes
   (a) Increase $600 by 24%
   (b) Increase $7200 by 5%
   (c) Decrease $95 by 10%
3. If 40% of the value of a house is $210,000, find the actual value of the house.
4. If 20% of Jack's wage is paid as rent, and the rent is $90 per week, what is his weekly wage?

Exercises 1.5 Percentages

1. (a) Express each fraction as a percentage:

   i. $\frac{87}{100}$ iii. $\frac{3}{4}$ v. $\frac{1}{3}$
   ii. $\frac{6}{10}$ iv. $\frac{4}{5}$ vi. $\frac{11}{21}$

   (b) Express each percentage as a fraction in its simplest form:

   i. 20% iii. 120% v. 13.5%
   ii. 66% iv. 75% vi. 6%

2. (a) Find 25% of \$800

   (b) What is 60% of 1 metre (in centimetres)?

   (c) What is $33\frac{1}{3}$% of \$9.30?

3. (a) In a clothes shop, jeans have a marked price of \$30, but a sign says 'Two for \$50 or 20% off the marked price'. What is the better deal if we want to buy two pairs?

   (b) Janet buys a can of drink which is labelled '33% free'. Assuming that this means 33% of the can (which contains 500ml), how much of the drink will she actually be paying for?

   (c) In the suburb of Redfield, there are 2000 people. The creative-arts area employs 125 of the population. What percentage of the population is this?

   (d) In 1994 there were 45 thousand spectators at the Gay and Lesbian Mardi Gras. How many watched the parade in 1995 if there was an attendance increase of 15%?

Answers 1.5

Section 1

1. (a) $\frac{4}{7}$
   (b) $\frac{3}{7}$
2. (a) $\frac{12}{25}$ (b) $\frac{13}{25}$ (c) $\frac{4}{25}$ (d) $\frac{21}{25}$

Section 2

1. (a) 90% (b) 20% (c) $37\frac{1}{2}\%$ (d) 60% (e) 85%
2. (a) $\frac{6}{25}$ (b) $\frac{3}{5}$ (c) $\frac{9}{20}$ (d) $\frac{3}{20}$ (e) $\frac{17}{200}$
3. (a) 0.64 (b) 0.08 (c) 0.215 (d) 0.19 (e) 0.024

Section 3

1. (a) 280 mL (b) \$5.2 (c) 1920 mL
2. (a) \$744 (b) \$7560 (c) \$85.5
3. \$525,000
4. \$450

Exercises 1.5

1. (a) i. 87% ii. 60% iii. 75% iv. 80% v. $33\frac{1}{3}\%$ vi. 52.38%
   (b) i. $\frac{1}{5}$ ii. $\frac{33}{50}$ iii. $1\frac{1}{5}$ iv. $\frac{3}{4}$ v. $\frac{27}{200}$ vi. $\frac{3}{50}$
2. (a) \$200 (b) 60 cm (c) \$3.10
3. (a) 20% off is cheaper
   (b) 335 ml
   (c) 6.25%
   (d) 51750 people