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Test One

This is a self-diagnostic test. Every pair of questions relates to a worksheet in a series available in the MUMS the WORD series. For example question 5 relates to worksheet 1.5 *Percentages*. If you score 100% then we feel you are adequately prepared for your introductory statistics course. For those of you who had trouble with a few of the questions, we recommend working through the appproriate worksheets and associated computer aided learning packages in this series.

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1 (a) $3 \times (2+5) - 6 \div 3$
  (b) $3 \times 3 + 2 \times 1 + 4 \div 2$

2 (a) What are the prime factors of 60?
  (b) What is the biggest number that divides 27 and 36?

3 (a) Simplify $\frac{10}{25}$.
  (b) Calculate $\frac{3}{8} + \frac{3}{10}$.

4 (a) Write $\frac{1}{20}$ as a decimal.
  (b) Write 0.06 as a fraction.

5 (a) Find 10 percent of 75.
  (b) Write $\frac{3}{5}$ as a percentage of a whole.

6 (a) Calculate $-8 + 10$.
  (b) Calculate $2 - 6$.

7 (a) Calculate -4×-3.
  (b) Calculate $-6/3$.

8 (a) Simplify $3^3 \times 2$.
  (b) Simplify $\sqrt{2} \times \sqrt{8}$.

9 (a) Simplify $x - (5 + x)$.
  (b) Simplify $\frac{x}{3} + \frac{x}{2}$.

10 (a) Expand and collect like terms:
  $(x+y)(x-y)$.
  (b) If $x = 2$ and $y = 5$, what is the value of $3x + 2y$?

Worksheet 1.1 Order of Operations

Section 1 BIDMAS

When performing arithmetic operations there is a particular order in which the operations must be done. If we fail to do things in the correct order, we may end up with the wrong answer. The order of operations is:-

1. Brackets
2. Indices
3. Division and multiplication
4. Addition and subtraction

Some people remember this as BIDMAS (an acronym for the above list). This means that when looking at a calculation you should first do the operations inside brackets, then do any division or multiplication, working from left to right in the expression. Finally do addition and subtraction, again working from left to right in the expression. Powers should be treated as a high-priority multiplication.

Example 1 :

What is $3 + 3 \times 10$?

By BIDMAS we do the multiplication first, hence

$$\begin{aligned} 3 + 3 \times 10 &= 3 + 30 \\ &= 33 \end{aligned}$$

Note: If we did not follow the correct order of operations we may have got 60 as our answer.

Example 2 : What is $3 \times (7 + 2) + 6$?

$$\begin{aligned} 3 \times (7 + 2) + 6 &= 3 \times 9 + 6 \quad \text{(do the bracket first)} \\ &= 27 + 6 \quad \text{(then do the multiplication)} \\ &= 33 \end{aligned}$$

Example 3 : What is $3 + 15 \div (\frac{1}{2} \times 6) - 9 \div 3^2$?

$$\begin{aligned} 3 + 15 \div (\frac{1}{2} \times 6) - 9 \div 3^2 &= 3 + 15 \div 3 - 9 \div 9 \quad \text{(do brackets and indices first)} \\ &= 3 + 5 - 1 \quad \text{(then do division)} \\ &= 7 \quad \text{(then work from left to right)} \end{aligned}$$

Example 4 : Work out $\frac{27}{3+6}$.

Note that 27 must be divided by (3+6), so we introduce brackets in the expression.

$$\begin{aligned} \frac{27}{3+6} &= 27 \div (3+6) \\ &= 27 \div 9 \\ &= 3 \end{aligned}$$

If you try to do the calculation without the brackets on your calculator, you will get an answer of 15. Why?

Exercises:

1. Calculate the following, without the use of a calculator:
   (a) $3 + 4 \times 7$
   (b) $6 + (7 \times 3) - (6 + 4)$
   (c) $18 \div (4 + 2)$
   (d) $6 + 3^2 - 4 \times 2$
   (e) $\frac{48}{6+2}$
   (f) $12 - \frac{1}{2} \times 10 + 4^2 \div 2$

Worksheet 1.2 Factorization of Integers

Section 1 Factors

When we multiply two numbers a and b together we get what is called the product of a and b. We call a and b the factors or divisors of ab. Many numbers have more than two factors, for instance:

We have $6 \times 2 = 12$ and $3 \times 4 = 12$. So 2 and 6 are factors of 12 and so are 3 and 4.

Factors come in pairs. When we wish to find all the factors of a number we do it systematically. We test to find all the numbers that divide evenly into the number we are trying to find the factors of.

Example 1 : Find the factors of 15.

1 clearly divides evenly into 15: $15 \times 1 = 15$. So 1 and 15 are factors of 15.

2 doesn't divide 15 evenly, so 2 is not a factor of 15.

3 goes evenly into 15 ($5 \times 3 = 15$) so 3 and 5 are factors of 15.

4 doesn't divide 15 evenly, so 4 is not a factor of 15.

5 was found to be a factor when we investigated 3. Since factors come in pairs, anything paired with a number bigger than 5 will be smaller than three, and we have already looked at these. When a number is found to be a factor, as 5 is, and its pair (in this case, 3) is smaller than the number you are looking at, all the factors have been found.

The factors of 15 are then: 1,3,5,15.

3 and 5 are called proper factors of 15. 1 and 15 are not proper factors since 1 divides everything evenly.

Example 2 : Find all the factors of 36.

$$\begin{aligned} 1 \times 36 &= 36 \\ 2 \times 18 &= 36 \\ 3 \times 12 &= 36 \\ 4 \times 9 &= 36 \\ 6 \times 6 &= 36 \end{aligned}$$

After 6×6 the answer will be smaller than the divisor (why?) so the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

Any number can be represented by its factors; for instance if I had \$20 I could also say I had four lots of \$5. Factors are useful in at least two ways:

1. Simplifying complicated expressions.
2. Simplifying fractions.

Example 3 :

$$\frac{10}{15} = \frac{2 \times 5}{3 \times 5}$$

We can cancel the 5 from the top and the bottom to get

$$\frac{10}{15} = \frac{2}{3}$$

We can say that $\frac{10}{15}$ and $\frac{2}{3}$ are equivalent fractions.

There will be more about simplifying fractions in another worksheet.

Exercises:

1. Find the factors of each of the following numbers:
   (a) 18
   (b) 24
   (c) 56
   (d) 128

Section 2 Prime Factorization

There is a special group of numbers called primes. These are numbers which have no proper factors. In other words the only numbers that divide evenly into primes are 1 and themselves. The first few primes are 2, 3, 5, 7, 11 and 13. The list goes on indefinitely.

Note: Primes are numbers which have no proper factors.

There are three types of whole numbers:

1. Composites: this is the name we give to numbers which have proper factors.
2. Primes: numbers which do not have proper factors.
3. Units: 1 is a unit.

In the previous section we saw how to represent composite numbers by the products of their factors. 12 may be represented in several ways:

$$12 = 6 \times 2 = 3 \times 4 = 3 \times 2 \times 2$$

$3 \times 2 \times 2$ is called the prime factorization of 12 since all the factors are prime. The prime factorization of a number is the product of primes which represent the number. Note that the 2 appears twice; we need this to actually represent 12 otherwise we would have $2 \times 3 = 6$ which is not equal to 12. The prime factorization of a number is unique. This means that there is only one way of writing a number as a product of primes (if you don't count the order it's written in).

Example 1 : Find the prime factorization of

- 7 $\quad 7 = 7$ since 7 is a prime.
- 45 $\quad 45 = 9 \times 5 = 3 \times 3 \times 5$
- 32 $\quad 32 = 2 \times 16 = 2 \times 2 \times 8 = 2 \times 2 \times 2 \times 4 = 2 \times 2 \times 2 \times 2 \times 2$

So, how do we find the prime factorization of a number? This is similar to finding the factors of a number. Test to see if the smaller primes divide evenly into the number first.

Example 2 : Find the prime factorization of 180

Is 2 a factor of 180? Yes: $180 = 2 \times 90$.

Is 2 a factor of 90? Yes: $90 = 2 \times 45$ and $180 = 2 \times 2 \times 45$.

Is 2 a factor of 45? No.

Is 3 a factor of 45? Yes: $45 = 3 \times 15$. So $180 = 2 \times 2 \times 3 \times 15$.

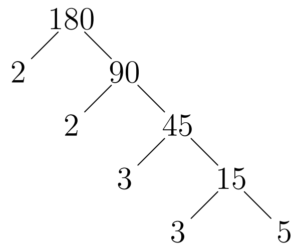
Is 3 a factor of 15? Yes: $15 = 3 \times 5$. So $180 = 2 \times 2 \times 3 \times 3 \times 5$.

5 is a prime, so we are finished.

$$180 = 2 \times 2 \times 3 \times 3 \times 5$$

Note: If we had tested 3 first we still would have ended up with the same factorization. It is important to make sure that you list all the primes the correct number of times because we use the prime factorization to represent the number.

Another way of finding the prime factorizations of a number is by using a factor tree. Here is a factor tree for 180.



When you use a prime number, stop and go to the other branch of the tree. So the prime factorization of 180 is

$$180 = 2 \times 2 \times 3 \times 3 \times 5$$

Exercises:

1. Find the prime factorization of each of the following numbers:
   (a) 84
   (b) 72
   (c) 126
   (d) 480
   (e) 26

Section 3 Highest Common Factors

A *common factor* of two numbers is a number which divides both of them (in other words a number which is a factor of both of them).

The *highest common factor* is the largest number which divides both numbers. The highest common factor is also called the *greatest common divisor*.

The factors of 12 are $1, 2, 3, 4, 6$ and 12. The factors of 18 are $1, 2, 3, 6, 9$ and 18. The highest common factor is the biggest number that appears in both lists. In this case, the answer is 6.

Note : To find the highest common factor of two numbers you can find all the factors of the smaller number; don't forget the number itself and, starting from the largest, test to see if they divide evenly into the bigger number.

Example 1 : Find the highest common factor of 72 and 180.

72 has factors $1, 2, 4, 6, 8, 9, 12, 18, 36$ and 72.

Does 72 go evenly into 180? No.

Does 36 go evenly into 180? Yes.

So 36 is the highest common factor of 72 and 180.

Example 2 : Find the highest common factor of 7 and 28.

7 has factors 1 and 7 only.

Does 7 go evenly into 28? Yes.

So 7 is the highest common factor of 7 and 28.

Example 3 : Find the highest common factor of 5 and 32.

5 is prime so its only factors are 1 and 5.

Does 5 go evenly into 32? No.

Clearly 1 will go evenly into 32, so 1 is the highest common factor of 5 and 32.

These last two examples illustrate what happens when one of the numbers is prime. If the larger number is prime the highest common factor will be 1 and if the smaller number is prime the highest common factor will either be 1 or the smaller number itself.

Exercises:

1. Find the highest common factor of these numbers:

    (a) 12 and 18
    (b) 15 and 45
    (c) 24 and 148
    (d) 56 and 242
    (e) 17 and 63

Section 4 Lowest Common Multiple

Given any two numbers we can find the smallest number which is a multiple of each of them; in other words we can find the smallest number which has both of them as factors. This number is called the lowest common multiple of the two numbers. The lowest common multiple is not always just the two numbers multiplied together.

> Example 1 : Find the lowest common multiple of 9 and 15.
>
> We are looking for a number which has 9 and 15 as factors.
>
> Multiples of 9 are $9, 18, 27, 36, 45, \dots$
>
> Multiples of 15 are $15, 30, 45, 60, \dots$
>
> Notice that 45 is the smallest number which occurs in both lists, so 45 is the lowest common multiple of 9 and 15. It is not $9 \times 15 = 135$ as we might have expected.

You could write down lists to find the lowest common multiple as was done in example 1, but the lists might get very long before you get a number in common. An easier way is outlined below.

First we find the highest common factor of the two numbers as outlined in the last section. So for our example above the highest common factor of 9 and 15 is 3.

Now we multiply 3 by the uncommon factors, i.e. the missing partners of 3 from the factorization of the initial two numbers. From our example we have $9 = 3 \times 3$ and $15 = 3 \times 5$, so the missing partners are 3 and 5. When we multiply these together with the highest common factor we get the lowest common multiple. So $3 \times 3 \times 5 = 45$.

Lets see if we can see why this works.

$$\begin{array}{rcl@{\qquad}c@{\qquad}rcl}
9 &=& 3 \times 3 & \text{and} & 15 &=& 3 \times 5 \\
45 &=& \overbrace{3 \times 3}^{9} \times 5 & & 45 &=& 3 \times \overbrace{3 \times 5}^{15} \\
45 &=& 9 \times 5 & & 45 &=& 3 \times 15
\end{array}$$

So the factors of both 9 and 15 appear in the lowest common multiple because of the way we set it up. They appear in the factorization only once because we only added the common factor once.

> Example 2 : Find the lowest common multiple of 180 and 72.

The highest common factor of 72 and 180 is 36 (as we found from the previous section).

$$
\begin{aligned}
72 &= 36 \times 2 \quad \text{and} \\
180 &= 36 \times 5 \\
\text{LCM} &= \overbrace{36 \times 2}^{72} \times 5 \\
\text{LCM} &= 2 \times \overbrace{36 \times 5}^{180} \\
360 &= 36 \times 2 \times 5 \\
360 &= 36 \times 5 \times 2
\end{aligned}
$$

Example 3 : Find the lowest common multiple of 32 and 5.

The highest common factor of 32 and 5 is 1.

$$
\begin{aligned}
32 &= 1 \times 32 \\
5 &= 1 \times 5 \\
\text{LCM} &= 1 \times 32 \times 5 \\
\text{LCM} &= 160
\end{aligned}
$$

Example 4 : Find the lowest common multiple of 18, 24 and 36. The highest common factor of 18, 24, and 36 is 6:

$$
\begin{aligned}
18 &= 6 \times 3 \\
24 &= 6 \times 2 \times 2 \\
36 &= 6 \times 2 \times 3
\end{aligned}
$$

Note that one of the missing partners is 3 and is repeated in the factors of 18 and 36. We include it once only in the factors of the LCM.

$$
\text{LCM} = \overbrace{6 \times 3}^{18} \times 4
$$

Therefore the LCM is 72.

Exercises:

1. Find the lowest common multiple of each of the following pairs of numbers:
   (a) 24 and 30
   (b) 18 and 32
   (c) 27 and 93

Exercises for Worksheet 1.2

1. (a) Is 4 a factor of 36? Explain your answer.
   (b) Is 5 a factor of 36? Explain your answer.
   (c) Write down the factors of

      i. 10 ii. 7 iii. 25 iv. 100

   (d) Find the prime factorization of

      i. 18 ii. 315

   (e) Factorize completely

      i. 126 ii. 144 iii. 100 iv. 63

   (f) Find the highest common factor of

      i. 5 & 6
      ii. 12 & 45
      iii. 120 & 63
      iv. 33 & 99

   (g) Find the lowest common multiple of

      i. 5 & 6
      ii. 12 & 45
      iii. 120 & 63
      iv. 33 & 99

2. (a) Is every number a factor of itself?
   (b) Is there a number which is a factor of all numbers?
   (c) What two numbers are factors of all even numbers?
   (d) Write down two numbers which have 2 and 5 as factors.
   (e) Suppose that 35 is a factor of a number. What two other numbers must be factors of the same number?
   (f) A number has factors including 2 and 3. What other number must also be a factor of the same number?
   (g) True or false? The lowest common multiple of 5 and 7 is the smallest number divisible by both 5 and 7?
   (h) True or false? A common multiple of two numbers is always divisible by both the numbers.
   (i) True or false? The lowest common multiple of two numbers is divisible by the highest common factor of the two numbers.
   (j) True or false? The lowest common multiple of 6 and 8 is 48.

Worksheet 1.3 Fractions

Section 1 Simplifying Fractions

Fractions arise often in everyday life. We use them when shopping, when cooking and when building. Numeric fractions have the form

$$\text{fraction} = \frac{\text{numerator}}{\text{denominator}}$$

where the numerator and the denominator are usually whole numbers. Numeric fractions are also called rational numbers. Notice that

$$b = \frac{b}{1}$$

where b is any number. So all numbers can be expressed as fractions in this way. Indeed there is more than one way to represent any fraction. Fractions which represent the same quantity in different ways are called *equivalent fractions*.

For instance $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}$ are equivalent fractions since they are all different ways of writing one half.

Indeed

$$\frac{x}{y} = \frac{x \times n}{y \times n}$$

where x, y and n are any numbers. This is because

$$\begin{aligned} \frac{n}{n} &= 1 \\ \text{and} \quad \frac{x \times n}{y \times n} &= \frac{x}{y} \times \frac{n}{n} \\ &= \frac{x}{y} \times 1 \\ &= \frac{x}{y} \end{aligned}$$

We use equivalent fractions in the arithmetic of fractions. Fractions in their simplest form have the property that the numerator and denominator have no common factors. See worksheet 1.2 to find out about factors if you need to.

To simplify fractions we break the numerator into factors and the denominator into factors and then we cancel common factors to leave us with an equivalent fraction in its simplest form.

Example 1 :

$$\begin{aligned}\frac{12}{24} &= \frac{12\times 1}{12\times 2}\\ &= \frac{12}{12}\times\frac{1}{2}\\ &= 1\times\frac{1}{2}\\ &= \frac{1}{2}\end{aligned}$$

Example 2 :

$$\begin{aligned}\frac{36}{15} &= \frac{12\times 3}{5\times 3}\\ &= \frac{12}{5}\times\frac{3}{3}\\ &= \frac{12}{5}\end{aligned}$$

Example 3 :

$$\begin{aligned}\frac{30}{45} &= \frac{6\times 5}{9\times 5}\\ &= \frac{2\times \cancel{3}\times \cancel{5}}{3\times \cancel{3}\times \cancel{5}}\\ &= \frac{2}{3}\end{aligned}$$

Note: A line through common factors in the numerator and the denominator helps to keep a track of working and is called canceling.

Exercises:

1. Simplify the following fractions

   (a) $\frac{8}{18}$
   (b) $\frac{16}{24}$
   (c) $\frac{27}{57}$
   (d) $\frac{180}{244}$
   (e) $\frac{256}{3250}$

Section 2 Mixed Numbers and Improper Fractions

A mixed number is one which has both an integer part and a fractional part, for instance $2\frac{1}{2}$ is a mixed number. When these appear in calculations you need to convert them to an equivalent improper fraction to make the calculations easier to perform. An improper fraction is one in which the numerator is larger than the denominator. This means that the fraction is bigger than 1. To convert a mixed number to an improper fraction, take the whole number part and replace it with an equivalent fraction with denominator the same as the fractional part. Then add this fraction to the fractional part to get the equivalent improper fraction.

Example 1 :

$$\begin{aligned} 2\frac{1}{2} &= \frac{2}{1} + \frac{1}{2} \\ &= \frac{2}{1} \times \frac{2}{2} + \frac{1}{2} \\ &= \frac{4}{2} + \frac{1}{2} \\ &= \frac{4+1}{2} \\ &= \frac{5}{2} \end{aligned}$$

Example 2 :

$$\begin{aligned} 1 + \frac{6}{7} &= \frac{1}{1} + \frac{6}{7} \\ &= \frac{1}{1} \times \frac{7}{7} + \frac{6}{7} \\ &= \frac{7}{7} + \frac{6}{7} \\ &= \frac{13}{7} \end{aligned}$$

The improper fractions so obtained can then be treated as any other fraction in the calculations performed.

Example 3 : Changing an improper fraction to a mixed numeral.

$$\begin{aligned}\frac{18}{5} &= \frac{15}{5}+\frac{3}{5}\\ &= 3+\frac{3}{5}\\ &= 3\frac{3}{5}\end{aligned}$$

Exercises:

1. Change the following mixed numerals to improper fractions
   (a) $1\frac{2}{3}$
   (b) $5\frac{3}{8}$
   (c) $2\frac{1}{4}$
   (d) $3\frac{2}{5}$

2. Change the following improper fractions to mixed numerals
   (a) $\frac{18}{7}$
   (b) $\frac{24}{9}$
   (c) $\frac{5}{3}$
   (d) $\frac{46}{5}$

Section 3 Multiplication and division of fractions

Multiplication is the simplest of fraction operations since

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

That is

$$\text{fraction}\times\text{fraction} = \frac{\text{product of numerators}}{\text{product of denominators}}$$

The only complication is remembering to cancel any common factors; this is easier if you simplify as much as possible before performing the multiplication.

Example 1 :

$$\begin{aligned}\frac{5}{9}\times\frac{6}{7} &= \frac{5\times 6}{9\times 7}\\ &= \frac{5\times \cancel{3}\times 2}{3\times \cancel{3}\times 7}\\ &= \frac{10}{21}\end{aligned}$$

Example 2 :

$$\begin{aligned}\frac{1}{2}\times\frac{4}{7} &= \frac{1\times 4}{2\times 7}\\ &= \frac{1\times 2\times \cancel{2}}{\cancel{2}\times 7}\\ &= \frac{2}{7}\end{aligned}$$

Example 3 :

$$\begin{aligned}\frac{13}{3}\times\frac{9}{26} &= \frac{13\times 9}{3\times 26}\\ &= \frac{\cancel{13}\times \cancel{3}\times 3}{\cancel{3}\times 2\times \cancel{13}}\\ &= \frac{3}{2}\end{aligned}$$

When multiplying mixed numbers, change the mixed numeral(s) to an improper fraction before multiplying.

Example 4 :

$$\begin{aligned}1\frac{2}{3}\times 2\frac{1}{4} &= \frac{5}{3}\times\frac{9}{4}\\ &= \frac{15}{4}\\ &= 3\frac{3}{4}\end{aligned}$$

Example 5 :

$$\begin{aligned}2\frac{3}{5}\times 1\frac{5}{13} &= \frac{13}{5}\times\frac{18}{13}\\ &= \frac{18}{5}\\ &= 3\frac{3}{5}\end{aligned}$$

We now look at division of fractions. The reciprocal of any rational number x is $\frac{1}{x}$. It is the thing you need to multiply x by to get 1. The reciprocal of a fraction $\frac{a}{b}$ is again the thing you need to multiply it by to get 1 and since

$$\frac{a}{b}\times\frac{b}{a}=\frac{a\times b}{b\times a}=1$$

the reciprocal of any fraction is given by its reversal. So the numerator becomes the denominator and the denominator becomes the numerator.

Example 6 : The reciprocal of $\frac{1}{2}$ is $\frac{2}{1} = 2$.

Example 7 : The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$.

Example 8 : The reciprocal of $3 = \frac{3}{1}$ is $\frac{1}{3}$.

Example 9 : The reciprocal of $2\frac{2}{3} = \frac{8}{3}$ is $\frac{3}{8}$. (That is, to find the reciprocal of a mixed number, we need to change it to a proper fraction).

Division of fractions makes use of the reciprocal. Dividing by a fraction is the same as multiplying by its reciprocal. So

$$\frac{a}{b} \div \frac{x}{y} = \frac{a}{b} \times \frac{y}{x}$$

To see why let's do an example.

$$\begin{aligned}
\frac{a}{b} \div \frac{x}{y} &= \frac{\frac{a}{b}}{\frac{x}{y}} \\
&= \frac{\frac{a}{b}}{\frac{x}{y}} \times \frac{\frac{y}{x}}{\frac{y}{x}} \\
&= \frac{\frac{a}{b} \times \frac{y}{x}}{\frac{x}{y} \times \frac{y}{x}} \\
&= \frac{\frac{a}{b} \times \frac{y}{x}}{1} \\
&= \frac{a}{b} \times \frac{y}{x}
\end{aligned}$$

Note: There is no need to do all this in normal working. You can skip from $\frac{a}{b} \div \frac{x}{y}$ to $\frac{a}{b} \times \frac{y}{x}$ straight away.

Example 10 :

$$\begin{aligned}
\frac{5}{8} \div \frac{1}{4} &= \frac{5}{8} \times \frac{4}{1} \\
&= \frac{5 \times 4}{2 \times 4} \\
&= \frac{5}{2} \\
&= 2\frac{1}{2}
\end{aligned}$$

Example 11 :

$$\begin{aligned}
\frac{(\frac{6}{7})}{(\frac{3}{7})} &= \frac{6}{7} \div \frac{3}{7} \\
&= \frac{6}{7} \times \frac{7}{3} \\
&= \frac{\cancel{3} \times 2 \times \cancel{7}}{\cancel{7} \times \cancel{3}} \\
&= 2
\end{aligned}$$

Example 12 :

$$\begin{aligned}
2\frac{1}{2} \div 1\frac{3}{5} &= \frac{5}{2} \div \frac{8}{5} \\
&= \frac{5}{2} \times \frac{5}{8} \\
&= \frac{25}{16} \\
&= 1\frac{9}{16}
\end{aligned}$$

Exercises:

1. Perform the following multiplications

   (a) $\frac{3}{5} \times \frac{7}{9}$

   (b) $\frac{13}{14} \times \frac{6}{10}$

   (c) $1\frac{2}{5} \times 2\frac{4}{7}$

   (d) $4\frac{1}{2} \times 1\frac{5}{21}$

2. Find the reciprocal of these numbers

(a) $\frac{2}{3}$

(b) $1\frac{1}{2}$

(c) $5\frac{2}{7}$

(d) $\frac{1}{8}$

3. Perform the following divisions

(a) $\frac{3}{5} \div \frac{6}{7}$

(b) $\frac{6}{11} \div \frac{9}{20}$

(c) $3\frac{1}{4} \div 1\frac{1}{2}$

(d) $8\frac{7}{12} \div 3\frac{3}{4}$

Section 4 Addition and Subtraction of Fractions

The addition and subtraction of fractions is slightly more complicated than the multiplication and division of them. The reason for this is that to add or subtract fractions, both fractions must have the same denominator. Given two fractions with the same denominator we do the addition and subtraction of the numerators and leave the denominator unchanged. If, for instance, we were adding three quarters and two quarters, the answer would be five quarters. To write this in a maths sentence we write

$$\frac{3}{4} + \frac{2}{4} = \frac{3+2}{4}$$

More generally, we would write

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \text{and} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Once you have the same denominator you make one fraction with the operation applying only to the numerator.

<u>Example 1</u> : $\frac{1}{2} + \frac{3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$

<u>Example 2</u> : $\frac{5}{12} + \frac{2}{12} = \frac{5+2}{12} = \frac{7}{12}$

<u>Example 3</u> : $\frac{6}{7} - \frac{1}{7} = \frac{6-1}{7} = \frac{5}{7}$

So what can we do when we are asked to calculate $\frac{1}{3} + \frac{1}{2}$? We need to take each fraction in the expression and replace it with an equivalent fraction, with the denominator of each equivalent fraction the same. To find which equivalent fractions are required we first find the lowest common multiple of the denominators. See worksheet 1.2 if you are unsure about lowest common denominators.

So for $\frac{1}{3}+\frac{1}{2}$ we look for the lowest common multiple of 2 and 3. This is 6. We now replace $\frac{1}{3}$ with an equivalent fraction which has denominator 6. We repeat this process for $\frac{1}{2}$.

$$\frac{1}{2}\times\frac{3}{3}=\frac{3}{6} \qquad \text{and} \qquad \frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$$

Therefore,

$$\frac{1}{3}+\frac{1}{2}=\frac{2}{6}+\frac{3}{6}=\frac{2+3}{6}=\frac{5}{6}$$

Example 4 :

$$\begin{aligned}\frac{1}{3}+\frac{1}{4} &= \frac{1}{3}\times\frac{4}{4}+\frac{1}{4}\times\frac{3}{3}\\ &= \frac{4}{12}+\frac{3}{12}\\ &= \frac{4+3}{12}\\ &= \frac{7}{12}\end{aligned}$$

Example 5 :

$$\begin{aligned}\frac{1}{5}+\frac{3}{10} &= \frac{1}{5}\times\frac{2}{2}+\frac{3}{10}\\ &= \frac{2}{10}+\frac{3}{10}\\ &= \frac{2+3}{10}\\ &= \frac{5}{10}\\ &= \frac{1}{2}\end{aligned}$$

Example 6 :

$$\begin{aligned}\frac{5}{8}-\frac{5}{12} &= \frac{5}{8}\times\frac{3}{3}-\frac{5}{12}\times\frac{2}{2}\\ &= \frac{15}{24}-\frac{10}{24}\\ &= \frac{15-10}{24}\\ &= \frac{5}{24}\end{aligned}$$

Example 7 :

$$\begin{aligned}
1\frac{2}{3}+3\frac{1}{4} &= 1+\frac{2}{3}+3+\frac{1}{4} \\
&= 4+\frac{2}{3}\times\frac{4}{4}+\frac{1}{4}\times\frac{3}{3} \\
&= 4+\frac{8}{12}+\frac{3}{12} \\
&= 4\frac{11}{12}
\end{aligned}$$

Example 8 :

$$\begin{aligned}
2\frac{4}{5}+1\frac{5}{8} &= 2+\frac{4}{5}+1+\frac{5}{8} \\
&= 3+\frac{4}{5}\times\frac{8}{8}+\frac{5}{8}\times\frac{5}{5} \\
&= 3+\frac{32}{40}+\frac{25}{40} \\
&= 3+\frac{57}{40} \\
&= 3+1\frac{17}{40} \\
&= 4\frac{17}{40}
\end{aligned}$$

Example 9 :

$$\begin{aligned}
6\frac{5}{8}-4\frac{1}{4} &= 6+\frac{5}{8}-\left(4+\frac{1}{4}\right) \\
&= 6+\frac{5}{8}-4-\frac{1}{4}\times\frac{2}{2} \\
&= 6-4+\frac{5}{8}-\frac{2}{8} \\
&= 2+\frac{3}{8} \\
&= 2\frac{3}{8}
\end{aligned}$$

Example 10 :

$$\begin{aligned}
4\frac{1}{7} - 1\frac{3}{5} &= 4 + \frac{1}{7} - \left(1 + \frac{3}{5}\right) \\
&= 4 + \frac{1}{7} \times \frac{5}{5} - 1 - \frac{3}{5} \times \frac{7}{7} \\
&= 4 + \frac{5}{35} - 1 - \frac{21}{35} \\
&= 3 + \frac{5}{35} - \frac{21}{35} \\
&= 2 + 1 + \frac{5}{35} - \frac{21}{35} \\
&= 2 + \frac{35}{35} + \frac{5}{35} - \frac{21}{35} \\
&= 2 + \frac{40}{35} - \frac{21}{35} \\
&= 2\frac{19}{35}
\end{aligned}$$

Exercises:

1. Evaluate the following additions and subtractions

   (a) $\frac{2}{3} + \frac{4}{7}$

   (b) $\frac{5}{8} - \frac{1}{3}$

   (c) $1\frac{1}{2} + 2\frac{4}{5}$

   (d) $8\frac{1}{4} + 2\frac{7}{9}$

   (e) $9\frac{5}{8} - 2\frac{1}{4}$

   (f) $7\frac{2}{3} - 1\frac{8}{9}$

   (g) $5\frac{1}{4} - 1\frac{1}{2}$

Exercises for Worksheet 1.3

1. (a) Write these fractions in their simplest form.

   i. $\frac{4}{10}$ ii. $\frac{21}{35}$ iii. $\frac{45}{54}$

   (b) Find the lowest common denominator for each pair of fractions.

   i. $\frac{1}{2}, \frac{2}{5}$ ii. $\frac{1}{3}, \frac{3}{5}$ iii. $\frac{1}{3}, \frac{3}{4}, \frac{3}{9}$

   (c) Change these mixed numbers into improper fractions

   i. $2\frac{3}{7}$ ii. $6\frac{3}{8}$ iii. $4\frac{5}{12}$

   (d) Change these improper fractions to mixed numbers

   i. $\frac{56}{9}$ ii. $\frac{27}{6}$ iii. $\frac{37}{4}$

   (e) Find the reciprical of

   i. $\frac{1}{3}$ ii. 5 iii. $\frac{4}{9}$

   (f) Put each group of numbers in ascending size

   i. $\frac{1}{5}, \frac{7}{5}, \frac{3}{5}$
   ii. $\frac{1}{10}, \frac{1}{100}, \frac{1}{5}, \frac{1}{20}$
   iii. $\frac{3}{8}, \frac{2}{3}, \frac{5}{12}$

2. Evaluate the following

   (a) $\frac{1}{8} + \frac{4}{8}$
   (b) $\frac{5}{6} + \frac{3}{4}$
   (c) $\frac{16}{25} - \frac{3}{5}$
   (d) $\frac{1}{2} - \frac{1}{3} + \frac{4}{9}$
   (e) $2\frac{1}{2} + 3\frac{1}{4}$
   (f) $\frac{3}{8} \times \frac{5}{7}$
   (g) $\frac{6}{7} \times \frac{14}{15}$
   (h) $\frac{1}{3} \div \frac{9}{5}$
   (i) $\frac{1}{100} \div \frac{1}{10}$
   (j) $10\frac{1}{2} \div 5\frac{1}{4}$
   (k) $3\frac{1}{3} \times 2\frac{3}{10} \times 12$
   (l) $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

3. (a) What fraction of one kilometre is one centimetre?

   (b) What fraction of one day is one second?

   (c) There are 16 girls in a class of 40. What fraction of the class is made up of boys?

   (d) Two family size pizzas (one ham and pineapple, and the other supreme) are cut into 10 and 12 pieces respectively. Bill eats 3 pieces of the ham and pineapple and Rowan eats 4 pieces of the supreme. Has Bill or Rowan eaten more pizza?

Worksheet 1.4 Fractions and Decimals

Section 1 Fractions to decimals

The most common method of converting fractions to decimals is to use a calculator. A fraction represents a division so $\frac{1}{a}$ is another way of writing $1 \div a$, and with a calculator this operation is easy to perform. The calculator will provide you with the decimal equivalent of the fraction.

Example 1 :

$$\frac{1}{2} = 1 \div 2 = 0.5$$

Example 2 :

$$\frac{7}{8} = 7 \div 8 = 0.875$$

Example 3 :

$$\frac{9}{8} = 9 \div 8 = 1.125$$

Example 4 :

$$\frac{5}{7} = 5 \div 7 = 0.7143$$

The last number has been rounded to four decimal places.

What is a decimal number?

This is a number which has a fractional part expressed as a series of numbers after a decimal point. It is a shorthand way of writing certain fractions. By first examining counting numbers we can then expand this thinking to include decimals.

The number 563 can be thought of as 5 hundreds + 6 tens + 3 ones. We can say there are 5 hundreds in 563; we read this information off from the hundreds column. There are 56 tens in 563 - there are 50 from the hundreds column and 6 from the tens column. There are 563 ones in 563.

The first place after the decimal point represents how many $\frac{1}{10}$'s there are in the number. The second place represents how many hundredths and the third place represents how many thousandths there are in the number.

Example 5 : 65.83 can be written as:

6 tens + 5 ones + 8 tenths + 3 hundredths.
There are 6 tens in 65.83
There are 65 ones in 65.83
There are 658 tenths in 65.83
and there are 6583 hundredths in 65.83

Example 6 : For the number 5.07 we have:

There are 5 ones in 5.07
There are 50 tenths in 5.07
There are 507 hundredths in 5.07

Note: This is different from 5.7. The zeros in a number matter.

Now we are in a position to say:

$$\begin{aligned} 0.1 &= \frac{1}{10} \\ 0.01 &= \frac{1}{100} \\ 0.001 &= \frac{1}{1000} \end{aligned}$$

And we can convert some fractions to decimals.

Example 7 :

$$\begin{aligned} \frac{3}{10} &= 0.3 \\ \frac{37}{100} &= 0.37 \\ \frac{37}{1000} &= 0.037 \\ \frac{37}{10} &= 3.7 \end{aligned}$$

Because the decimal places represent tenths, hundreths, etc, when we wish to convert a fraction to a decimal we use equivalent fractions with denominators of $10, 100, 1000$ etc. So our method of finding the decimal equivalent for many fractions is to find an equivalent fraction with the right denominator. We then use the information that

$$\begin{aligned}
\frac{1}{10} &= 0.1 \\
\frac{1}{100} &= 0.01 \\
\frac{1}{1000} &= 0.001
\end{aligned}$$

to convert it to a decimal number.

Example 8 :

$$\begin{aligned}
\frac{1}{4} = \frac{1}{4} \times \frac{25}{25} &= \frac{25}{100} \\
&= \frac{20}{100} + \frac{5}{100} \\
&= \frac{2}{10} + \frac{5}{100} \\
&= 0.25
\end{aligned}$$

$$\begin{aligned}
\frac{2}{5} = \frac{2}{5} \times \frac{2}{2} &= \frac{4}{10} \\
&= 0.4
\end{aligned}$$

$$\begin{aligned}
\frac{1}{8} = \frac{1}{8} \times \frac{125}{125} &= \frac{125}{1000} \\
&= 0.125
\end{aligned}$$

Sometimes it is not immediately obvious what number you need to multiply by to get the right denominator, but on the whole the ones you are asked to do without a calculator should be fairly simple.

Exercises:

1. Convert the following fractions to decimals

(a) $\frac{3}{10}$	(f) $\frac{1}{2}$
(b) $\frac{15}{100}$	(g) $\frac{3}{4}$
(c) $\frac{8}{1000}$	(h) $\frac{4}{5}$
(d) $\frac{367}{1000}$	(i) $\frac{85}{100}$
(e) $\frac{85}{10}$	(j) $\frac{19}{50}$

Section 2 Decimals to fractions

To convert decimals to fractions you need to recall that

$$\begin{aligned} 0.1 &= \frac{1}{10} \\ 0.01 &= \frac{1}{100} \\ 0.001 &= \frac{1}{1000} \end{aligned}$$

Thus the number after the decimal place tells you how many tenths, the number after that how many hundredths, etc. Form a sum of fractions, add them together as fractions and then simplify using cancellation of common factors.

Example 1 :

$$\begin{aligned} 0.51 &= \frac{5}{10} + \frac{1}{100} \\ &= \frac{50}{100} + \frac{1}{100} \\ &= \frac{51}{100} \end{aligned}$$

Example 2 :

$$\begin{aligned} 0.75 &= \frac{7}{10} + \frac{5}{100} \\ &= \frac{70}{100} + \frac{5}{100} \\ &= \frac{75}{100} \\ &= \frac{3}{4} \times \frac{25}{25} \\ &= \frac{3}{4} \end{aligned}$$

Example 3 :

$$\begin{aligned} .102 &= \frac{1}{10} + \frac{0}{100} + \frac{2}{1000} \\ &= \frac{100}{1000} + \frac{2}{1000} \\ &= \frac{102}{1000} \\ &= \frac{51}{500} \end{aligned}$$

Exercises:

1. Convert the following decimals to fractions

   (a) 0.7
   (b) 0.32
   (c) 0.104
   (d) 0.008
   (e) 0.0013

Section 3 Operations on decimals

When adding or subtracting decimals it is important to remember where the decimal point is. Adding and subtracting decimals can be done just like adding and subtracting large numbers in columns. The decimal points must line up. Fill in the missing columns with zeros to give both numbers the same number of columns. Remember that the zeros either go at the very end of numbers after the decimal point or at the very beginning before the decimal point.

Example 1 : Calculate $0.5 - 0.04$.

$$\begin{array}{l} 0.5\ - \\ 0.04 \end{array}$$

$$\begin{array}{l} 0.50\ - \\ \underline{0.04} \\ 0.46 \end{array}$$

- Line up the decimal points
- Insert zeros so that both numbers have the same number of digits after the decimal point
- Perform the calculation, keeping the decimal point in place

Example 2 :
Calculate $0.07 - 0.03$.

$$\begin{array}{l} 0.07\ - \\ \underline{0.03} \\ 0.04 \end{array}$$

Example 3 : Calculate $1.1 - 0.003$. This can be written as

$$\begin{array}{l} 1.1\ - \\ 0.003 \end{array}$$

$$\begin{array}{l} 1.100\ - \\ \underline{0.003} \\ 1.097 \end{array}$$

Exercises:

1. Evaluate without using a calculator
   (a) $0.72 + 0.193$
   (b) $0.604 - 0.125$
   (c) $0.8 - 0.16$
   (d) $32.104 + 41.618$
   (e) $54.119 - 23.24$

Exercises for Worksheet 1.4

1. (a) How many hundreds, tens, ones, tenths, hundredths etc. are there in the following?

   i. 60.31 ii. 704.2 iii. 14.296

   (b) Write the number represented by

   i. 6 hundreds, 4 ones, 9 hundredths
   ii. 8 tens, 2 thousandths
   iii. 9 ones, 2 tenths, 3 thousandths
   iv. $64 + \frac{1}{10} + \frac{4}{1000} + \frac{6}{10000}$
   v. $100 + 60 + 2 + \frac{1}{100} + \frac{9}{10000}$
   vi. $1000 + \frac{3}{10} + \frac{9}{100}$

2. (a) Change the following decimals into fractions without the use of a calculator.

   i. 0.2 iv. 0.12 vii. 6.04
   ii. 0.04 v. 0.639 viii. 0.625
   iii. 0.002 vi. 1.7 ix. 0.3

   (b) Change these fractions to decimals without the use of a calculator.

   i. $\frac{1}{10}$ iv. $\frac{402}{500}$ vii. $\frac{3}{4}$
   ii. $\frac{2}{100}$ v. $\frac{3}{20}$ viii. $\frac{1}{8}$
   iii. $\frac{27}{50}$ vi. $\frac{6}{25}$ ix. $\frac{3}{15}$

   (c) Using a calculator, convert the following fractions to decimals:

   i. $\frac{7}{8}$ ii. $\frac{1}{3}$ iii. $1\frac{2}{7}$

3. Evaluate the following without a calculator.

   (a) $0.62 - 0.37$
   (b) $0.08 + 0.2$
   (c) $6.72 + 6.1$
   (d) $0.675 + 0.21 + 0.008$
   (e) $4.70 - 0.356$
   (f) $6.32 - 2.8 + 1.01$

Worksheet 1.5 Percentages

Section 1 Proportions

A pie is cut into twelve pieces. John eats five pieces, Peter eats one piece and Chris and Michael eat three pieces each. If we ask what proportion of the pie John ate we are asking what fraction of the total pie he ate. The proportion John ate is $\frac{5}{12}$.

Proportions are comparisons, usually between part of something and the whole of it. Chris and Michael each had $\frac{3}{12}$ or $\frac{1}{4}$ of the pie.

For the proportion of pie John ate you could instead say that he ate 5 in 12 parts of the pie. This means for every twelve pieces of pie John ate five. To write it this way instead of the fractional way we write 5:12 and say five in twelve. Proportions can be simplified in the same way as fractions, i.e. by canceling common factors.

Example 1 : For every hundred students enrolled in first-year Maths at a university 55 of them are males. What proportion of first-year Maths students at the university are female?

$$\begin{aligned} 100 - 55 &= 45 \text{ females in every } 100 \\ \text{proportion of females} &= \frac{45}{100} \\ \frac{45}{100} &= \frac{9\times \cancel{5}}{20\times \cancel{5}} = \frac{9}{20} \end{aligned}$$

So the proportion of females in first-year Maths is $\frac{9}{20}$ or 9:20. Hence we could say that for every 20 students enrolled in the first year maths course, 9 of them are female.

Example 2 : In a week a car dealer sells 10 red cars, 8 blue cars, 20 white cars and 2 black cars. What proportion of cars sold were red? What proportion were not black? Well, $10 + 8 + 20 + 2 = 40$ cars were sold in a week. But 10 cars were red so the proportion of red cars sold is

$$\frac{10}{40} = \frac{1}{4}$$

which can also be denoted 1:4. The proportion of cars sold that were not black is

$$\frac{40-2}{40} = \frac{38}{40} = \frac{19}{20} \quad \text{or} \quad 19:20.$$

Exercises:

1. In a class of 28 students, 16 were boys.
   (a) What proportion of the students were boys?
   (b) What proportion of the class were girls?
2. A group of 50 people were interviewed, who worked in the CBD of Edge city. Of the 50: 24 traveled by bus to work, 8 by car, and 18 by train. What proportion
   (a) Traveled by bus
   (b) Did not travel by bus
   (c) Traveled by car
   (d) Traveled by bus or train

Section 2 Percentages and Decimals

Percentages are another way of talking about proportions. When comparing proportions you may end up with long lists of varying denominators, so it is simpler to standardise the denominator for comparing proportions. The standard denominator is 100 and a percentage is a number out of 100 (per cent meaning out of 100 in Latin). Thus 80% means a proportion of 80 in 100 or $\frac{80}{100}$.

To convert proportions to percentages, then, is a matter of finding an equivalent fraction with denominator 100. To review equivalent fractions see worksheet 1.3. The percentage is the numerator of a fraction which has denominator 100.

Example 1 : Express $\frac{4}{5}$ as a percentage.

$$\frac{4}{5} = \frac{4}{5} \times \frac{20}{20} = \frac{80}{100} = 80\%$$

Notice that we get the same answer if we do the calculation this way:

$$\frac{4}{5} = \frac{4 \times 100}{5} \times \frac{1}{100} = \frac{400}{5} \times \frac{1}{100} = 80 \times \frac{1}{100} = 80\%$$

So if we wish to convert a proportion to a percentage we can simply multiply by $\frac{100}{1}$ to get a percentage amount.

Example 2 : Express $\frac{3}{4}$ as a percentage.

$$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 75\%$$

Express $\frac{1}{8}$ as a percentage.

$$\frac{1}{8} \times \frac{100}{1} = \frac{100}{8} = 12.5\%$$

Since a percentage is the numerator of a fraction with a denominator of 100 they can also be expressed as decimals.

Example 3 :

$$\begin{aligned} 0.07 &= \frac{7}{100} = 7\% \\ 0.12 &= \frac{12}{100} = 12\% \\ 217\% &= \frac{217}{100} = 2.17 \end{aligned}$$

Exercises:

1. Convert the following to percentages:

   (a) $\frac{9}{10}$
   (b) $\frac{20}{100}$
   (c) $\frac{3}{8}$
   (d) $\frac{3}{5}$
   (e) $\frac{17}{20}$

2. Convert the following percentages to fractions, and simplify where necessary:

   (a) 24%
   (b) 60%
   (c) 45%
   (d) 15%
   (e) $8\frac{1}{2}\%$

3. Convert the following percentages to decimals

   (a) 64%
   (b) 8%
   (c) 21.5%
   (d) 19%
   (e) 2.4%

Section 3 Problems relating to Percentages

Often you will be asked to find a particular percentage of a quantity. For example you might need to find 20% of \$500. To do this you use multiplication of fractions. The percentage is expressed in its equivalent fraction form and you then multiply it by the quantity to get the answer.

Example 1 : Find 20% of \$500

$$\frac{20}{100} \times \frac{500}{1} = \frac{20 \times 5 \times 100}{100 \times 1} = 100$$

So 20% of \$500 is \$100.

Example 2 : A jacket in a shop costs \$60. It is marked down by 5%. How much will you pay for the jacket?

First find 5% of \$60 and then subtract the answer from the price of the jacket. Alternatively find 95% of \$60. This is how much you will pay for the jacket.

$$\frac{5}{100} \times \frac{60}{1} = \frac{300}{100} = 3$$

So 5% of \$60 is \$3. The jacket will sell for \$60-\$3=\$57.

Alternatively

$$\frac{95}{100} \times \frac{60}{1} = \frac{95 \times 60}{100} = 19 \times 3 = 57$$

The jacket will sell for \$57. That is, the jacket will sell for 95% of the original price.

You can choose either of the methods illustrated above to get the answer.

Example 3 : The price of a clock which costs \$80 is to be increased by 15%.

Method A: Find 15% of \$80 and add this amount to \$80.

$$\frac{15}{100} \times \frac{80}{1} = \frac{15}{5} \times \frac{4}{1} = \frac{60}{5} = 12$$

Hence %15 of \$80 is \$12. The price of the clock is increased by \$12 to \$92.

Method B: Find %115 of \$80. (%115 = %100 + %15).

$$\frac{115}{100} \times \frac{80}{1} = \frac{115}{5} \times \frac{4}{1} = \frac{460}{5} = 92$$

So the price of the clock is increased to \$92.

Some questions might give you a percentage with a corresponding amount and ask you to work out what the total quantity is. If you take the quantity given and divide by the percentage you get the quantity equivalent to 1%. Now multiply by 100 and you will have the amount corresponding to 100%.

Example 4 : A car is marked down to 75% of its original price. It now costs \$15000. What was its original price?

That is,

$$\begin{aligned}
75\%\text{of the original price} &= 15000 \\
1\%\text{of the original price} &= \frac{\text{Quantity}}{\text{Percentage}} \\
&= \frac{15000}{75} \\
&= \frac{150 \times 100}{75} \\
&= \frac{2 \times 100}{1} \\
&= 200
\end{aligned}$$

So \$200 is 1% of the original price. That is,

$$\begin{aligned}
1\%\text{of the original price} &= 200 \\
100\%\text{of the original price} &= 200 \times 100 \\
&= 20000
\end{aligned}$$

Therefore the original price of the car was \$20000.

Exercises:

1. Find the following percentages:
   (a) 40% of 700 ml
   (b) 65% of $8
   (c) 32% of 6L (put your answer in mL)
2. Perform the following changes
   (a) Increase $600 by 24%
   (b) Increase $7200 by 5%
   (c) Decrease $95 by 10%
3. If 40% of the value of a house is $210,000, find the actual value of the house.
4. If 20% of Jack's wage is paid as rent, and the rent is $90 per week, what is his weekly wage?

Exercises for Worksheet 1.5

1. (a) Express each fraction as a percentage:

   i. $\frac{87}{100}$
   ii. $\frac{6}{10}$
   iii. $\frac{3}{4}$
   iv. $\frac{4}{5}$
   v. $\frac{1}{3}$
   vi. $\frac{11}{21}$

   (b) Express each percentage as a fraction in its simplest form:

   i. 20%
   ii. 66%
   iii. 120%
   iv. 75%
   v. 13.5%
   vi. 6%

2. (a) Find 25% of \$800
   (b) What is 60% of 1 metre (in centimetres)?
   (c) What is $33\frac{1}{3}\%$ of \$9.30?

3. (a) In a clothes shop, jeans have a marked price of \$30, but a sign says 'Two for \$50 or 20% off the marked price'. What is the better deal if we want to buy two pairs?
   (b) Janet buys a can of drink which is labelled '33% free'. Assuming that this means 33% of the can (which contains 500ml), how much of the drink will she actually be paying for?
   (c) In the suburb of Redfield, there are 2000 people. The creative-arts area employs 125 of the population. What percentage of the population is this?
   (d) In 1994 there were 45 thousand spectators at the Gay and Lesbian Mardi Gras. How many watched the parade in 1995 if there was an attendance increase of 15%?

Worksheet 1.6 Signed Numbers

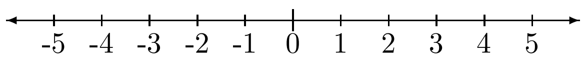
Section 1 Introduction to Signed Numbers

Signed numbers are positive and negative numbers. So far in the worksheets we have mainly talked about the counting numbers, which are all positive. In maths we call the set of these natural numbers $\mathbb{N}$. We have also talked about fractions. All fractions including whole numbers are in a set called the rationals and represented by the symbol $\mathbb{Q}$. If we include the negative numbers to the counting numbers we end up with the set of integers represented by $\mathbb{Z}$. The set of real numbers $\mathbb{R}$ includes positive and negative integers, fractions and irrational numbers.

If my cheque account was in credit \$30 I could say that the balance was +\$30. If it was overdrawn \$30 the balance would be -\$30. Temperatures are measured in degrees. In Sydney the average summer temperature is about $+27^o$. In Berlin the average winter temperature is about -10^o. In some way, we consider the negative of a number to be its opposite. If you were in debit \$30 and you made a deposit of \$30, i.e. you added \$30 to your account, you would have a zero balance. Since we consider $-b$ to be the opposite of b, what is $-(-b)$? It is the opposite of $-b$ which is b. This statement can be written as a rule:

Two negatives make a positive. Only in maths, of course.

When we want to picture the ordinary number system we can often think about it as a line which is infinitely long in both directions. We choose a point to be zero and the numbers to the right of zero are positive. The numbers to the left of zero are negative. The number line looks like this:



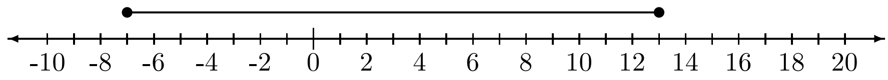
Fractions and other numbers can be fitted in on the number line. For instance $1\frac{1}{2}$ is half-way between 1 and 2. And $-1\frac{1}{2}$ is half-way between -1 and -2. If b is any point on the number line then $-b$ is the same distance from zero but on the opposite side of the number line.

Section 2 Operations involving signed numbers

Example 1 : My cheque account is overdrawn \$7, and I make a deposit of \$20. What is my new balance? It is \$13. This can be written as

$$-7 + 20 = 13$$

When we add a number, we go to the right on the number line:



Example 2 : The temperature outside in the morning is -15^o. During the day the temperature rises by 10^o. At the end of the day it is -5^o. This can be written as

$$-15 + 10 = -5$$

In example 1 we have

$$-7 + 20 = 20 - 7 = 13$$

In example 2 we have

$$-15 + 10 = -5 = -(15 - 10)$$

These two examples illustrate the general rules:

$$\begin{aligned} -a + b &= b - a \\ -a + b &= -(a - b) \end{aligned}$$

Note: These are just two ways of writing the same thing. Because $-(-a) = a$ we can write a positive sign or a plus sign as two minuses. Also recall that $a + b = b + a$, so we can change the order of an addition without affecting it. These rules will become familiar with practice.

Example 3 : My cheque account balance is \$13 and I write a cheque for \$25. My account becomes \$12 overdrawn. This can be written as

$$13 - 25 = -12$$

It is not long before I receive a letter from the bank informing me of a \$30 dishonour fee. Then the amount in arrears is

$$-12 - 30 = -42$$

Example 4 : The temperature in downtown London is -10^o during the day. At night it drops a further 5^o. This brings the temperature down to -15^o. We can write this as

$$-10 - 5 = -15$$

In example 3 we have

$$13 - 25 = -12 = -(25 - 13)$$

In example 4 we have

$$-10 - 5 = -15 = -(10 + 5)$$

These examples illustrate the following rules

$$\begin{aligned} b - a &= -(a - b) \\ -a - b &= -(a + b) \end{aligned}$$

These rules will also become familiar with practice.

The rearranging of these formulae to make things easier to calculate uses some basic rules of arithmetic. One mentioned earlier is that $a + b = b + a$. Another is that subtracting a positive number is the same as adding a negative number, so that

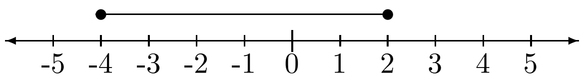
$$a - b = a + (-b)$$

And a final one is that a minus sign in front of a bracket means you should change the sign of everything inside the brackets when you remove them. So

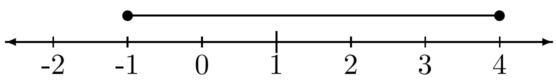
$$\begin{aligned} -(a - b) &= -a + b \\ -(-a + b) &= a - b \end{aligned}$$

Example 5 : Calculate the following using a number line:

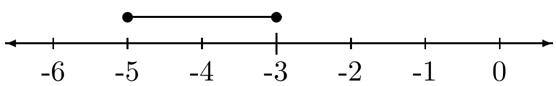
(a) $-4 + 6$ (Go to -4 on the number line, then go 6 places to the right).



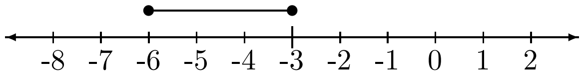
(b) $4 - 5$ (Go to 4 on the number line, then go 5 places to the left).



(c) $-3 - 2$ (Go to -3 on the number line, then go 2 places to the left).



(d) $-6 - (-3) = -6 + 3$ (Go to -6 on the number line, then go 3 places to the right).



Example 6 :

(a) $2-(-4)=2+4=6$

(b) $2-(-4-1)=2-(-5)=2+5=7$

(c) $-3+8=8-3=5$

(d) $-3-12=-(3+12)=-15$

(e) $-7+4=-(7-4)=-3$

(f) $\frac{5}{2}-\frac{6}{2}=\frac{5-6}{2}=\frac{-(6-5)}{2}=-\frac{1}{2}$

Exercises:

1. Using any of the methods given in this worksheet, but without the use of a calculator, work out the following:

(a)	$-3+5$	(b)	$15-20$
(c)	$-8-2$	(d)	$-10+14$
(e)	$-3-(-2)$	(f)	$-8+7$
(g)	$-4+4$	(h)	$-40-9$
(i)	$6-14$	(j)	$2-12$

Exercises for Worksheet 1.6

1. Evaluate the following:
   (a) $12 - 7$
   (b) $-12 + 7$
   (c) $-12 - 7$
   (d) $-4 + 12$
   (e) $-2 - (-1)$
   (f) $-(-3) + 7$
   (g) $10 - 2 - 3 - 5$
   (h) $-6 - 2 - 12$
   (i) $-6 - 3 + (-2)$

2. Evaluate the following:
   (a) $26.3 - 13.2$
   (b) $-14.02 + 11.7$
   (c) $-210.1 - 306.2$
   (d) $-27.5 - (-1.3)$
   (e) $79.6 + (-0.05)$
   (f) $61.03 - (9.1)$

3. (a) A bank account has an account balance of -$600. That is, it is $600 overdrawn. If there is a debit of $250, what is the new balance? If there has been a bank error, and the debit of $250 is cancelled, what is the balance now?
   (b) In a quiz show, you win a point for each correct answer, and lose a point for each incorrect answer. Dale gets 15 right and 20 wrong: what is her score? If the rules change, and it is decided that incorrect answers are ignored, what would Dale's score be now?
   (c) If the temperature is -4 degrees one morning, and 11 degrees the next, what is the temperature difference?
   (d) Joan went on a three-month diet. She lost 2 kg in the first month, 1.1 kg in the second, and 3.2 kg in the third. How much weight did she lose altogether? Write an expression, using negative numbers, to show this loss.
   (e) Peter starts at point A and walks 6 km south, then 17km north, then 2km south. How far is he from his starting point? Write an expression using negative and positive numbers; let north be positive and south be negative.

Worksheet 1.7 Further Signed Numbers

Section 1 Multiplication of signed numbers

Multiplication is a shorthand way of adding together a large number of the same thing. For example, if I have 3 bags of oranges with 5 oranges in each bag, then I have 15 oranges in total: $5 + 5 + 5 = 5 \times 3 = 15$. Notice that if instead I had 5 bags of oranges with 3 in each bag, then I would still have 15 oranges: $3 + 3 + 3 + 3 + 3 = 3 \times 5 = 15$.

This illustrates a mathematical property called commutitivity which says that the order of multiplication doesn't matter: $a \times b = b \times a$. All real numbers have this property. Notice that

$$-1 \times 5 = 5 \times -1 = -5$$

Because I can arrange numbers in any order when I multiply numbers that do not have the same sign, I can write

$$\begin{aligned} -5 \times 4 &= -1 \times 5 \times 4 \\ &= -1 \times 20 \\ &= -20 \end{aligned}$$

Similarly,

$$\begin{aligned} 5 \times -4 &= 5 \times 4 \times -1 \\ &= 20 \times -1 \\ &= -20 \end{aligned}$$

Now, to be able to multiply any two numbers of any sign, we need one more piece of information, and that is:

$$-1 \times -1 = 1$$

<u>Example 1</u> :

$$\begin{aligned} -5 \times -4 &= -1 \times 5 \times -1 \times 4 \\ &= -1 \times -1 \times 5 \times 4 \\ &= -1 \times -1 \times 20 \\ &= 1 \times 20 \\ &= 20 \end{aligned}$$

With a little thought we can come up with the following rules:

- multiplying numbers with like signs gives a positive number
- multiplying numbers with unlike signs gives a negative number

Exercises:

1. Perform the following multiplications:

   (a) -3×4

   (b) 7×-2

   (c) -16×-1

   (d) 8×4

   (e) -9×-4

   (f) $-12 \times \frac{1}{2}$

   (g) $-\frac{1}{4} \times -\frac{1}{2}$

   (h) $-27 \times -\frac{1}{3}$

   (i) -1.2×3

   (j) -1.47×-10

Section 2 Division of signed numbers

If I had a box of 60 apples, and I wanted to divide it evenly amongst 4 people, each person would get 15 apples. The mathematical formula that expresses this calculation is:

$$60 \div 4 = 15$$

Another way of looking at the apple situation is that I need to find one quarter of 60. The formula would then be written:

$$60 \times \frac{1}{4} = 15$$

These two formulae express the same thing. For any two numbers x and y (so long as $y \neq 0$),

$$x \div y = x \times \frac{1}{y}$$

Dividing by a number is the same thing as multiplying by the reciprocal of the number. The reciprocal of y is $\frac{1}{y} = 1 \div y$ for $y \neq 0$. This is true even if y is a fraction, or is negative (or both). Now we can deal with division of signed numbers in a similar way to the multiplication of signed numbers.

Example 2 :

$$\begin{aligned} 60 \div -4 &= 60 \times \frac{-1}{4} \\ &= 60 \times \frac{1}{4} \times -1 \\ &= 15 \times -1 \\ &= -15 \end{aligned}$$

Example 3 :

$$\begin{aligned} -60 \div -3 &= -60 \times \frac{-1}{3} \\ &= -1 \times 60 \times \frac{1}{3} \times -1 \\ &= -1 \times -1 \times 20 \\ &= 1 \times 20 \\ &= 20 \end{aligned}$$

Notice that the last expression could have been written:

$$\frac{-60}{-3} = \frac{-1}{-1} \times \frac{60}{3} = 1 \times 20$$

since the division of one number by itself is always 1.

An important thing to notice with signed fractions is that all of the following expressions are equivalent:

$$\frac{-6}{7} = -1 \times \frac{6}{7} = \frac{6}{-7}$$

The usual way to write this is with the minus sign on top, or sitting out in front of the entire fraction as in

$$\frac{-a}{b} = -\frac{a}{b}$$

As with multiplication, we have a rule for dividing signed numbers:

- dividing numbers with like signs gives a positive number
- dividing numbers with unlike signs gives a negative number

When doing divisions, it is important to keep in mind that division by zero is not defined.

Exercises:

1. Perform the following divisions:

   (a) $-8 \div -2$
   (b) $-20 \div -5$
   (c) $6 \div 3$
   (d) $18 \div 9$
   (e) $-36 \div -4$
   (f) $8 \div -\frac{1}{2}$
   (g) $-\frac{5}{8} \div -\frac{1}{4}$
   (h) $16 \div -\frac{1}{8}$
   (i) $-\frac{2}{5} \div \frac{7}{10}$
   (j) $2\frac{1}{4} \div -9$

Section 3 More multiplication and division

When confronted by a multiplication and/or division which has more than two numbers involved, we just extend the processes already described. Here's a couple of examples:

Example 1 :

$$\begin{aligned} -2 \times -3 \times 5 \times -2 &= -1 \times 2 \times -1 \times 3 \times 5 \times -1 \times 2 \\ &= -1 \times -1 \times -1 \times 2 \times 3 \times 5 \times 2 \\ &= 1 \times -1 \times 30 \times 2 \\ &= -1 \times 60 \\ &= -60 \end{aligned}$$

Alternatively, this may be done working from left to right:

$$\begin{aligned} -2 \times -3 \times 5 \times -2 &= (-2 \times -3) \times 5 \times -2 \\ &= 6 \times 5 \times -2 \\ &= 30 \times -2 \\ &= -60 \end{aligned}$$

Example 2 :

$$\begin{aligned} -2 \times -5 \times 6 \times 3 &= -1 \times 2 \times -1 \times 5 \times 6 \times 3 \\ &= -1 \times -1 \times 2 \times 5 \times 6 \times 3 \\ &= 1 \times 10 \times 18 \\ &= 180 \end{aligned}$$

This example may also be done working from left to right:

$$\begin{aligned} -2 \times -5 \times 6 \times 3 &= (-2 \times -5) \times 6 \times 3 \\ &= 10 \times 6 \times 3 \\ &= 60 \times 3 \\ &= 180 \end{aligned}$$

Notice that because $-1 \times -1 = 1$, when there are an even number of negative numbers the product will be positive and when there are an odd number of negative numbers, the product will be negative. This results holds for division as well, because we can rewrite any division as multiplication by the reciprocal.

Exercises:

1. Evaluate

   (a) $-3 \times 4 \times -5$
   (b) $6 \times -2 \times 3$
   (c) $-20 \times -4 \times -5$
   (d) $-2 \times -3 \times 2 \times -4$
   (e) $\frac{1}{2} \times 30 \times -2$
   (f) $40 \times -\frac{1}{2} \times -\frac{1}{4} \times 2$
   (g) $18 \div -\frac{1}{5}$
   (h) $-20 \div 2 \times \frac{1}{4}$
   (i) $800 \times -\frac{1}{20} \times -3 \times -\frac{1}{10}$
   (j) $(6 \times -\frac{1}{2}) \times (20 \div \frac{1}{4})$
   (k) $(30 \div -2) \div (-10 \div -2)$
   (l) $2\frac{1}{2} \times \frac{2}{7} \times -\frac{1}{5}$
   (m) $-8\frac{1}{4} \div -3 \times -\frac{2}{5}$

(n) $-\frac{3}{4} \times -\frac{2}{9} \times -4 \times -5$

(o) $16 \times \frac{1}{2} \times 3 \times 5$

(p) $\frac{2}{3} \times -1 \times 8 \div -2$

(q) $-1 \times 7 \times -3 \times -\frac{1}{4} \times 20$

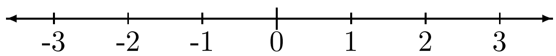
(r) $(-6 \times 4) \div (-8 \times \frac{1}{2})$

(s) $(20 \div -2) \times (-18 \div -9)$

(t) $(-1)^2 \times (-2)^3$

Section 4 Introduction to absolute values

When we write $|a|$, we are referring to the absolute value of a. We can think of the absolute value of a number as its distance away from zero. So if we look at the number line,



then we see that -3 is 3 units away from zero. So $|-3| = 3$. Absolute values are never negative.

Example 1 :

$$\begin{aligned} |-7| &= 7 \\ |6| &= 6 \\ |7| &= 7 \end{aligned}$$

If we are asked to work out the absolute value of a sum such as $|-a+b|$, we should treat the vertical bars as brackets in the sense that we do the operations inside the bars first. So

$$\begin{aligned} |-3+2| &= |-1| = 1 \\ |-5 \times 2 + 6| &= |-10+6| = |-4| = 4 \end{aligned}$$

Notice that we cannot generally write $|a+b| = |a|+|b|$. Try some a's and b's to satisfy yourself of this. Try making one of a or b negative.

Example 2 : If I went for a Sunday drive and went 30 kilometres east and then 15 kilometres west, I would be 15 kilometres from my starting point even though I had traveled 45 kilometres in total. This illustrates that we cannot take the absolute value of the inside bits separately. If we think of east as a positive direction, and west as a negative direction, then we have been considering the problem

$$\begin{aligned} |\text{Thirty km east} + \text{Fifteen km west}| &= |30 - 15| \\ &= |15| \\ &= 15 \end{aligned}$$

On the other hand, if we mistakenly consider the quantity as the absolute value of each bit:

$$\begin{aligned} |\text{Thirty km east}| + |\text{ Fifteen km west}| &= |30| + |-15| \\ &= 30 + 15 \\ &= 45 \end{aligned}$$

Therefore we can see that

$$|30 - 15| \neq |30| + |-15|$$

Exercises for Worksheet 1.7

1. (a) Which of the following have the same answer?

   i. 6×3
   ii. $+6 \times (-3)$
   iii. $-6 \times (+3)$
   iv. $-6 \times (-3)$
   v. $(+6) \times (+3)$
   vi. -6×3

   (b) Which are equivalent?

   i. $-6 \div +7$
   ii. $\frac{-6}{7}$
   iii. $\frac{6}{7}$
   iv. $-\frac{6}{7}$
   v. $\frac{-6}{-7}$
   vi. $-6 \div -7$

   (c) Evaluate the following:

   i. $30 \times (-2)$
   ii. $-25 \times (-4)$
   iii. $30 \div (-2)$
   iv. $(-32) \div (-8)$
   v. $-100 \div 25$
   vi. $(-2) \times 4 \times (-5)$
   vii. $(-6) \times (-3) \times (-7)$
   viii. $-6 \times (-3) + (2) \times 4$
   ix. $-5 + 4 \times (-7)$
   x. $-9 \div (-3) + 7 \times (-2)$
   xi. $6(-2) + 4(-3)$
   xii. $-2(-7) - (-3)(-4)$

   (d) Evaluate the following:

   i. $|-4|$
   ii. $|-2| + |3|$
   iii. $|-2+3|$
   iv. $|4| - |-4|$
   v. $|4 - (-4)|$
   vi. $|1 - |3||$

Worksheet 1.8 Power Laws

Section 1 Powers

In maths we sometimes like to find shorthand ways of writing things. One such shorthand we use is powers. It is easier to write 2^3 than $2 \times 2 \times 2$. The cubed sign tells us to take the number and multiply it by itself 3 times. The 3 is called the index. Then 10^6 means multiply 10 by itself 6 times. This means:

$$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$$

We can do calculations with this shorthand. Look at this calculation:

$$3^2 \times 3^3 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

because 3 is now being multiplied by itself 5 times. So we could have just written $3^3 \times 3^2 = 3^5$. The more general rule is

$$x^a \times x^b = x^{a+b}$$

where x, a and b are any numbers.

We add the indices when we multiply two powers of the same number.

Example 1 :

$$5^6 \times 5 = 5^7$$

Note that $5 = 5^1$.

Example 2 :

$$x^3 \times x^b = x^{3+b}$$

Example 3 :

$$3^3 \times 3^0 = 3^3$$

so that 3^0 must be equal to 1. Indeed, for any non zero number x, $x^0 = 1$.

$$\boxed{x^0 = 1 \qquad \text{if } x \neq 0}$$

We can only use this trick if we are multiplying powers of the same number. Notice that we can't use this rule to simplify $5^3 \times 8^4$, as the numbers 5 and 8 are different.

This shorthand in powers gives us a way of writing $(3^2)^3$. In words, $(3^2)^3$ means: take 3, multiply it by itself, then take the result, and multiply that by itself 3 times. Then

$$(3^2)^3 = (3 \times 3)^3 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) = 3^6$$

The general form of the rule in multiplying powers is

$$\boxed{(x^a)^b = x^{a \times b}}$$

<u>Example 4</u> :

$$\begin{aligned} (5^2)^4 &= 5^8 \\ (x^3)^b &= x^{3 \times b} \end{aligned}$$

Finally, what happens if we have different numbers raised to powers? Say we have $3^2 \times 5^3$. In this particular case, we would leave it as it is. However, in some cases, we can simplify. One case is when the indices are the same. Consider $3^2 \times 6^2$. Then

$$\begin{aligned} 3^2 \times 6^2 &= 3 \times 3 \times 6 \times 6 \\ &= 3 \times 6 \times 3 \times 6 \\ &= (3 \times 6)^2 \\ &= 18^2 \end{aligned}$$

We can get the second line because multiplication is commutative, which is to say that $a \times b = b \times a$. The general rule then when the indices are the same is

$$\boxed{x^a \times y^a = (x \times y)^a}$$

<u>Example 5</u> :

$$2^2 \times 3^2 \times 5^2 = (2 \times 3 \times 5)^2 = 30^2$$

Exercises:

1. Simplify the following and leave your answers in index form:

   (a) $6^3 \times 6^7$
   (b) $4^5 \times 4^2$
   (c) $x^7 \times x^9$
   (d) $m^4 \times m^3$
   (e) $(m^4)^3$
   (f) $(8^2)^3$
   (g) $5^3 \times 5^9$
   (h) $x^6 \times x^{12} \times x^3$
   (i) $(x^3)^4 \times x^5$
   (j) $m^4 \times (m^5)^2 \times m$

Section 2 Negative Powers

We can write $\frac{1}{x}$ as x^{-1}. That is: $x^{-1} = \frac{1}{x}$. Now we can combine this notation with what we have just learnt.

Example 1 :

$$\begin{aligned} \frac{1}{x \times x \times x \times x} &= \frac{1}{x^4} \\ &= (x^4)^{-1} \\ &= x^{-4} \end{aligned}$$

Example 2 :

$$2^{-3} = (2^3)^{-1} = 8^{-1} = \frac{1}{8}$$

We treat negative indices in calculations in the same manner as positive indices. Then

$$\begin{aligned} x^b \times x^{-a} &= x^{b+(-a)} = x^{b-a} \\ (x^b)^{-a} &= x^{-ab} \\ x^{-n} &= \frac{1}{x^n} \end{aligned}$$

Consider this longhand example:

Example 3 :

$$\begin{aligned} 2^{-3} \times 2^5 &= \frac{1}{2 \times 2 \times 2} \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} \\ &= 2 \times 2 \\ &= 2^2 \end{aligned}$$

whereas our shorthand notation gives: $2^{-3} \times 2^5 = 2^{-3+5} = 2^2$.

This concept may be written in the form of a division.

Example 4 : $x^7 \times \frac{1}{x^6} = x^7 \div x^6$. When we divide two powers of the same number, we subtract the indices. Hence,

$$\boxed{x^m \div x^n = x^{m-n}}$$

So

$$\begin{aligned} x^7 \div x^6 &= x^{7-6} \\ &= x^1 \\ &= x \end{aligned}$$

Example 5 :

$$\begin{aligned} 6^8 \div 6^3 &= 6^{8-3} \\ &= 6^5 \end{aligned}$$

Example 6 :

$$\begin{aligned} m^4 \div m^9 &= m^{4-9} \\ &= m^{-5} \\ &= \frac{1}{m^5} \end{aligned}$$

Example 7 :

$$\begin{aligned} x^8 \div x^{-2} &= x^{8-(-2)} \\ &= x^{8+2} \\ &= x^{10} \end{aligned}$$

Exercises:

1. Simplify the following and leave your answers in index form:

   (a) $6^{-4} \times 6^7$

   (b) $10^8 \times 10^{-5}$

   (c) $x^7 \times x^3$

   (d) $(x^{-2})^3$

   (e) $y^{-12} \times y^5$

   (f) $y^8 \div y^3$

   (g) $7^2 \div 7^{-4}$

   (h) $(m^4)^{-2} \times (m^3)^5$

   (i) $y^6 \times y^{14} \div y^5$

   (j) $(8^3)^4 \div (8^2)^3$

Section 3 Fractional Powers

What do we mean by $4^{\frac{1}{2}}$? The notation means that we are looking for a number which, when multiplied by itself, gives 4. Then $4^{\frac{1}{2}} = 2$ because $2 \times 2 = 4$. In general, $x^{\frac{1}{a}}$ is asking us to find a number which, when multiplied by itself a times, gives us x. In the case when the index is $\frac{1}{2}$, as above, we also use the square-root sign: $x^{\frac{1}{2}} = \sqrt{x}$. So $8^{\frac{1}{3}}$ means the number which when multiplied by itself 3 times gives us 8. That is, $8^{\frac{1}{3}}$ is the cube root of 8, and is written as $8^{\frac{1}{3}} = \sqrt[3]{8}$.

$$8^{\frac{1}{3}} = 2 \text{ because } 2 \times 2 \times 2 = 8$$

What about $8^{\frac{2}{3}}$? With our previous rule about powers, we end up with this calculation:

$$8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2 = (2)^2 = 4$$

Example 1 :

$$8^{\frac{1}{3}} \times 8^{\frac{2}{3}} = 8^{\frac{1}{3}+\frac{2}{3}} = 8^{\frac{3}{3}} = 8^1 = 8$$

And, if we have $8^{\frac{1}{2}} \times 2^{\frac{1}{2}}$, because the indices are the same, then we can multiply the numbers together. Then

$$8^{\frac{1}{2}} \times 2^{\frac{1}{2}} = (8 \times 2)^{\frac{1}{2}} = 16^{\frac{1}{2}} = 4$$

Another way of writing this is

$$\sqrt{8} \times \sqrt{2} = \sqrt{8 \times 2} = \sqrt{16} = 4$$

The simplification process can often be taken only so far with simple numbers. Consider

$$5^{\frac{1}{3}} \times 3^{\frac{1}{3}} = (5 \times 3)^{\frac{1}{3}} = 15^{\frac{1}{3}}$$

There is no simpler way of writing $15^{\frac{1}{3}}$, so we leave it how it stands.

Example 2 : Recall that $x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}}$. Then

$$\begin{aligned} 9^{-\frac{1}{2}} &= \frac{1}{9^{\frac{1}{2}}} \\ &= \frac{1}{3} \end{aligned}$$

Exercises:

1. Simplify the following:
   (a) $9^{\frac{1}{2}}$
   (b) $27^{\frac{1}{3}}$
   (c) $16^{\frac{1}{2}}$
   (d) $16^{-\frac{1}{2}}$
   (e) $27^{-\frac{2}{3}}$
2. Rewrite the following in index form:
   (a) $\sqrt{8}$
   (b) $\sqrt[3]{m}$
   (c) $(m^6)^{\frac{1}{2}}$
   (d) $(10^{\frac{1}{2}})^3$
   (e) $(16^{\frac{1}{2}})^{-2}$

Exercises for Worksheet 1.8

1. (a) Express $3 \times 3 \times 3 \times 2 \times 2$ using powers.
   (b) Write 27 in index form using base 3.
   (c) Calculate the following. Which are the same?

      i. $2^2 \times 3^2$
      ii. $2^2 + 3^2$
      iii. $(2+3)^2$
      iv. $(2 \times 3)^2$
      v. $\frac{2^2}{3^2}$
      vi. $(\frac{2}{3})^2$

   (d) Express $\frac{1}{5^2}$ in index form with base 5.
   (e) Express $\frac{1}{27}$ in index form with base 3.
   (f) Express $\sqrt{64}$ in index form with base 64.

2. Simplify the following:

   (a) $2^3 \times 2^4$
   (b) $(3^2)^5$ (leave in index form)
   (c) $12^5 \div 12^7$
   (d) $(2.3)^2(2.3)^{-4}$ (leave in index form)
   (e) $8^{-\frac{1}{3}}$
   (f) $\frac{4^8}{4^{12}} \times 4^{-3}$ (leave in index form)
   (g) $\frac{2^1}{2^{-3}} + (2^2 + 2^1)^2$
   (h) $(0.01)^2$
   (i) $10^5 \div (3^2 \times 10^{-2})^3$ (leave in index form)

Worksheet 1.9 Introduction to Algebra

Section 1 Algebraic Expressions

Algebra is a way of writing arithmetic in a general form. You have already come across some algebraic expressions in previous worksheets. An algebraic expression is one in which the arithmetic is written with symbols rather than numbers. The most common use of algebra is in writing formulae. A formula is an algebraic expression which acts as a general 'recipe':

Example 1 :

$$C = 2.54 \times I$$

where I represents the number of inches, and C represents the number of centimetres. This formula represents a recipe for converting a length in inches to one in centimetres. If a ruler, say, is 12 inches long, then we can use the formula to work out how long it is in centimetres. It is $2.54 \times 12 = 30.48$cm.

Example 2 : You want to buy tickets for a show for 2 adults and 2 children. Let the price for an adult ticket be a (in dollars) and the price for a child c (again in dollars). Then the total cost P is

$$P = a + a + c + c$$

If the adult's tickets are \$55, and the children's tickets are \$30, then we would have

$$\begin{aligned} a &= 55 \\ c &= 30 \end{aligned}$$

so that $P = 55 + 55 + 30 + 30$. But if you realize that $P = 2a + 2b = 2(a + b)$ then you have an easier calculation to do, and it is also easy to substitute in other prices for the tickets.

Here are some algebraic expressions that we have already seen in the worksheets:

1. ab means a multiplied by b
2. $(-a)b = -ab$ means $-a$ multiplied by b
3. $2(x + y)$ means the sum of x and y all multiplied by 2
4. $2x + y$ means y added to 2 lots of x

5. $xx = x^2$ means x multiplied by itself

6. $\frac{p}{q}$ and p/q both mean p divided by q

7. $\frac{a}{x+y}$ means a divided by the sum of x and y

Note that multiplication signs are often omitted or replaced by brackets. When we multiply two numbers, say 3 and 5, we must write 3×5 or $(3)5$ rather than 35, which is indistinguishable from the number thirty-five. Remember that, when we are dealing with numbers, symbols represent numbers. This means that $a \times b = b \times a$ and that $(ab)c = a(bc) = abc$.

Section 2 Simplifying Algebraic Expressions

In many algebraic expressions we look for ways of simplifying, or tidying up, the expression so that it appears in its most compact form. In our previous example about the price of tickets,

$$P = 2(a + c)$$

is much neater than

$$P = a + a + c + c$$

The first step in many such simplifications is to collect like terms. The terms in an algebraic expression are the parts that are separated by + and − signs. For instance, in the expression

$$5a + 3c + 2d - 7a$$

the terms are $5a$, $3c$, $2d$ and $7a$. The terms which have exactly the same letters in them are called *like terms*.

Example 1 :
In the expression

$$7xy - 3x + 2xy + 4x - 5y$$

$7xy$ and $2xy$ are like terms and $-3x$ and $4x$ are also like terms. Our expression can be simplified as follows:

$$\begin{aligned} 7xy - 3x + 2xy + 4x - 5y &= 7xy + 2xy + 4x - 3x - 5y \\ &= 9xy + x - 5y \end{aligned}$$

Example 2 :
In the expression

$$5x^2 - 2x + 7x^2$$

the terms are $5x^2$, $2x$, and $7x^2$; the like terms are $5x^2$ and $7x^2$. The whole expression can be simplified:

$$\begin{aligned} 5x^2 - 2x + 7x^2 &= 5x^2 + 7x^2 - 2x \\ &= 12x^2 - 2x \end{aligned}$$

Collecting like terms means to bring them together as a single term; an example of this was replacing $a + a$ with $2a$ with the ticket example. Then $3x^3 + 5x^3$ can be replaced with $8x^3$. Notice how the sign in front of the term remains with the term, and, where there is no sign, a positive term is implied. Then $2x - 6x = -4x$.

Example 3 :

$$\begin{aligned} 5x^2 + 3x + 2x^2 - x &= 5x^2 + 2x^2 + 3x - x \\ &= 7x^2 + 2x \end{aligned}$$

Example 4 :

$$\begin{aligned} \frac{1}{3}x + \frac{1}{4}x &= (\frac{1}{3} + \frac{1}{4})x \\ &= (\frac{4}{12} + \frac{3}{12})x \\ &= \frac{7}{12}x \end{aligned}$$

Exercises:

1. Simplify

   (a) $3x - 2x + 4x$

   (b) $-xy + 2xy + x$

   (c) $2x^2y + x^2 - y^2 + 3y^2$

   (d) $\frac{1}{2} - x^2 - \frac{1}{3}x + x^2$

   (e) $5 + x - 3 + y + 6x + 2$

   (f) $xyz + yz + xz - 3yz$

Section 3 Removal of Brackets

Whatever is inside brackets should be treated as a single term. If you have an expression which involves only addition and subtraction there are two rules to remember.

1. An addition sign, or plus sign, in front of the brackets leaves the sign of every term inside the brackets unchanged.
2. A subtraction sign in front of the bracket indicates that, when removing the bracket, the sign of all terms inside must be changed.

Then

$$\begin{aligned} -(a+b) &= -a-b \\ -(a-b) &= -a+b \\ -(-a-b) &= a+b \\ (-a+b) &= -a+b \end{aligned}$$

If there is no sign before the brackets, a positive sign is implied as is the case with other terms. The removal of brackets is called expanding.

Example 1 :

$$\begin{aligned} -(x+2)+x &= -x-2+x \\ &= = -x+x-2 \\ &= -2 \end{aligned}$$

Often an algebraic expression will be simplified by expanding the bracketed terms and collecting terms.

Example 2 :

$$\begin{aligned} 3+5x-(2x+3)+5y &= 3+5x-2x-3+5y \\ &= 3x+5y \end{aligned}$$

Exercises:

1. Simplify
   (a) $-(x+2)+(x-1)$
   (b) $x^2+x-(x^2-x)$
   (c) $\frac{1}{2}(x+y)-\frac{1}{2}(x-y)$
   (d) $2x^2+y^2-\frac{1}{4}(x^2+y^2)$
2. Simplify
   (a) $(x+1)^2+(x-1)^2$
   (b) $(2x+y)^2-(2x+y)$
   (c) $x^2-(x+y)^2$
   (d) $\frac{1}{4}x^2+(\frac{x+1}{2})^2$

Exercises for Worksheet 1.9

1. Simplify the following:

   (a) $2a + 5a$

   (b) $6a + 2b - 3a$

   (c) $6x - 7x + 8x^2$

   (d) $-(x + y) + 2x$

   (e) $-(x - y) + y$

   (f) $4ab + 6a - 2ab - 4a$

   (g) $3x + (x - y) + y$

   (h) $2x^3 - 3x^2 + 2x^2 - 7$

   (i) $(-p + q) - (-p - q)$

   (j) $\frac{1}{3}x + \frac{1}{4}y - \frac{2}{3}x - \frac{3}{8}y$

2. (a) Jason wants to buy p books at \$15 each. Write a mathematical sentence to find the total cost C of the books.

   (b) I intend buying b avocados at 90 cents each and r rockmelons at \$1.20 each. Write a mathematical sentence to find the total cost C of the purchase.

   (c) Alex has m children and Yvonne has n children. Write a mathematical sentence to find the total number of children, N, that they have between them.

   (d) Greg lives 4 times as far from the university as Jerry. If Jerry lives w kilometres away, write an expression to show the number of kilometres, t, that Greg lives from the university.

Worksheet 1.10 Further Algebra

Section 1 Distributive Laws

When collecting like terms, we need to know things like

$$\begin{aligned} 2(a+b) &= 2a+2b \\ \text{and } 3(a-b) &= 3a-3b \end{aligned}$$

This is called the distributive law, which is a rule that holds true in the arithmetic of numbers. In general

$$a(b+c) = ab+ac$$

Example 1 :

$$\begin{aligned} 5(x+2y) &= 5x+10y \\ 3x(x+y) &= 3x^2+3xy \\ -2(a+b) &= -2a-2b \\ -3(a-b) &= -3a+3b \end{aligned}$$

These are just extensions of the rules for removing brackets when an addition or subtraction sign is in front of the brackets. Here are some examples that involve collecting like terms:

$$\begin{aligned} x(y+2)-3(y+2x) &= xy+2x-3y-6x \\ &= xy-3y+2x-6x \\ &= xy-3y-4x \\ a(b+c)+b(a-c) &= ab+ac+ba-bc \\ &= ab+ac+ab-bc \\ &= 2ab+ac-bc \end{aligned}$$

Notice that, in the second example, we also used the commutative law: $ab = ba$.

Exercises:

1. Expand the following by removing the brackets:

   (a) $6(x+3)$

   (b) $2(3x-4)$

(c) $-3(x+y)$

(d) $5(m-4)$

(e) $x(x+y)$

(f) $y(y^2-2)$

(g) $t(3t+4)$

(h) $4(y-5)$

(i) $-x(x+2)$

(j) $-3(m-n)$

2. Remove the brackets and collect like terms in each of the following:

(a) $2(m+4)+3(m+6)$

(b) $4(t-2)-3(t+1)$

(c) $7(m-3)-2(m-4)$

(d) $-4(x+1)+3(x+2)$

(e) $5(2t+1)+3(t+2)$

(f) $4(x-3)+2(5-x)$

(g) $x(x+4)+x(x-3)$

(h) $t(t+1)-4(t+2)$

(i) $m(m-4)+2(m+1)$

(j) $3(x+2)+4x$

Section 2 Further Distributive Laws

Example 1 : Sometimes we need to expand and simplify algebraic expressions like

$$(x+2)(x+3)$$

If we think of $(x+3)$ as a single term for the time being, and call it A, so that $A=x+3$, then

$$\begin{aligned}
(x+2)(x+3) &= (x+2)A \\
&= xA+2A && \text{by the distributive law} \\
&= x(x+3)+2(x+3) && \text{substituting for } A \\
&= x^2+3x+2x+6 \\
&= x^2+5x+6 && \text{collecting like terms}
\end{aligned}$$

Example 2 :

$$\begin{aligned}(x-5)(y+2) &= (x-5)B \qquad \text{if} \quad B = y+2 \\ &= xB - 5B \\ &= x(y+2) - 5(y+2) \\ &= xy + 2x - 5y - 10\end{aligned}$$

When we are doing these calculations we normally multiply each term in the first bracket by each term in the second and leave out the middle steps. However, putting in all the steps at first is a good idea.

Example 3 :

$$\begin{aligned}(y+2)(y-1) &= y(y-1) + 2(y-1) \\ &= y^2 - y + 2y - 2 \\ &= y^2 + y - 2\end{aligned}$$

To expand expressions which have one set of brackets inside another, just follow the rules already given, although it is best to do the inner-most brackets first then the outer ones.

Example 4 :

$$\begin{aligned}(x + (5-y)2) + 5(x - 3(y+1)) &= (x + 10 - 2y) + 5(x - 3y - 3) \\ &= x + 10 - 2y + 5x - 15y - 15 \\ &= x + 5x - 2y - 15y + 10 - 15 \\ &= 6x - 17y - 5\end{aligned}$$

Note that it is the convention for the constant term, in this case -5, to be put last in the expression when writing an answer, and for the other terms to be written in alphabetical order.

Exercises:

1. Expand the following and collect like terms:

   (a) $(x+2)(x+5)$
   (b) $(x+4)(x+1)$

(c) $(y+2)(y-3)$

(d) $(m+7)(m-5)$

(e) $(x+8)(x-3)$

(f) $(2x+1)(x+2)$

(g) $(3m+2)(m-4)$

(h) $(x+3)(x-3)$

(i) $(y+5)(y-5)$

(j) $(m+5)^2$

2. Expand the following and collect like terms:

   (a) $(2x+3(x+1))+4(x+2)$

   (b) $(8x-2(x+3))+3(x+4)$

   (c) $(2x+4(x+1))+2(x+3(x+4))$

   (d) $(2m+3(m+4))-2(m+1)$

   (e) $2(4m-6)-5(2m+3(m+1))$

Section 3 Introduction to Substitution

Algebraic expressions are useful to us as a way of representing quantities without assigning explicit numerical values. For example, the perimeter of a rectangle is twice the length plus twice the width. Let the length be represented by l, the width by w, and the perimeter by P. Then we have the formula

$$P = 2l + 2w$$

We can use this formula to find the perimeter of any rectangle, once the width and length are given. If a rectangle is 5cm long and 2 cm wide, then the perimeter is

$$P = 2 \times 5 + 2 \times 2 = 10 + 4 = 14$$

so the perimeter is 14 cm.

Example 1 :
If $x = 4$ and $y = 3$, what does $xy + 5x$ equal?
Well, $xy + 5x = 4 \times 3 + 5 \times 4 = 12 + 20 = 32$.

Example 2 :
What does $5(x+y)+3(x+z)$ equal when $x=1$ and $y=1$?

$$\begin{aligned} 5(x+y)+3(x+z) &= 5(1+1)+3(1+z) \\ &= 5\times 2+3+3z \\ &= 10+3+3z \\ &= 13+3z \end{aligned}$$

Sometimes, we are given an algebraic equation and asked to find what the value of a variable must be for the equation to be true. We will discuss this more fully in Worksheet 12, but here is a simple example.

Example 3 :
If $x=6$, and $2x+y=13$, what does y equal? We substitute the known value of x into the other equation and have the following working:

$$\begin{aligned} 2x+y &= 13 \\ 2\times 6+y &= 13 \\ 12+y &= 13 \end{aligned}$$

Clearly, $y=1$.

Exercises:

1. Given that $x=4$, find the value of:
   (a) $2x$
   (b) $x-7$
   (c) $3x+1$
   (d) x^2
   (e) $\sqrt{x}$
   (f) $x^3+\sqrt{x}$
2. Given that $x=5$ and $y=3$, find the value of:
   (a) $x+y$
   (b) $2y+30$
   (c) $3(x+2y)$

(d) $8y - 4x$

(e) $2(2x - y)$

(f) $x^2 + y^2$

(g) $2x - y + 7$

(h) $8(2x + 3(y - 4))$

(i) $(x + 2)(y + 4)$

(j) $(2x - 4) + 3(y - 2)$

Exercises for Worksheet 1.10

1. Simplify by removing brackets.

   (a) $3(p + q)$

   (b) $3a(a + 4)$

   (c) $-4a(b - 6)$

   (d) $x + 2(x - 3)$

   (e) $5(x + y) - 2(x - y)$

   (f) $2x(x + 1) + 3(x + 1)$

   (g) $(x + 3)(x - 2)$

   (h) $(3x - 2)(2x - 9)$

   (i) $(x + 3(2 - y)) + 5(x - (y - 1))$

   (j) $((2x + 1)(x + 2))(x + 3)$

2. (a) If $m = x + y$, find m when $x = 4$ and $y = 8$.

   (b) If $I = Prt$, find I when $P = 1000$, $r = 0.01$ and $t = 4$.

   (c) If $P = 2x + 2y$, find P when $x = 10$ and $y = 12$.

   (d) If $s = \frac{2(D-X)}{T}$, and you are given that $D = 60$, $T = 2$, and $X = 40$, find s.

   (e) If $c = \frac{AY}{Y+12}$ is the formula for a child's dose of medicine, where A is the adult dose in grams and Y is the child's age in years, find the dose for a 10-year-old if the adult dose is 50 grams.

   (f) What does $2(p - q) + 3(p + q)$ equal when p is 7 and q is -2?

Answers to Test One
and
Exercises from Worksheets 1.2 - 1.10

Answers to Test One

1. (a) 19 (b) 13
2. (a) 2, 3, 5 (b) 9
3. (a) $\frac{2}{5}$ (b) $\frac{27}{40}$
4. (a) 0.05 (b) $\frac{6}{100}$
5. (a) 7.5 (b) 60%
6. (a) 2 (b) -4
7. (a) 12 (b) -2
8. (a) 54 (b) 4
9. (a) -5 (b) $\frac{5x}{6}$
10. (a) $x^2 - y^2$ (b) 6

Worksheet 1.1

Section 1

1. (a) 31 (b) 17 (c) 3 (d) 7 (e) 6 (f) 15

Worksheet 1.2

Section 1

1. (a) 1, 2, 3, 6, 9, 18
   (b) 1, 2, 3, 4, 6, 8, 12, 24

(c) 1, 2, 4, 7, 8, 14, 28, 56

(d) 1, 2, 4, 8, 16, 32, 64, 128

Section 2

1. (a) $84 = 2 \times 2 \times 3 \times 7$
   (b) $72 = 2 \times 2 \times 2 \times 3 \times 3$
   (c) $126 = 2 \times 3 \times 3 \times 7$
   (d) $480 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$
   (e) $26 = 2 \times 13$

Section 3

1. (a) 6 (b) 15 (c) 4 (d) 2 (e) 1

Section 4

1. (a) 120 (b) 32×9 (c) 31×9

Exercises 1.2

1. (a) Yes, as $4 \times 9 = 36$
   (b) No as multiples of 5 end in 0 or 5.
   (c) i. 10, 5, 2, 1
       ii. 7, 1
       iii. 25, 5, 1
       iv. 100, 50, 25, 20, 10, 5, 4, 2, 1
   (d) i. $18 = 2 \times 3 \times 3$
       ii. $315 = 3 \times 3 \times 5 \times 7$
   (e) i. $126 = 2 \times 3 \times 3 \times 7$
       ii. $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
       iii. $100 = 2 \times 2 \times 5 \times 5$
       iv. $63 = 3 \times 3 \times 7$

(f)
i. 1
ii. 3
iii. 3
iv. 33

(g)
i. 30
ii. 180
iii. 2520
iv. 99

2. (a) Yes, though not a *proper* factor
(b) Yes, 1
(c) 1 and 2
(d) 10 and 20
(e) 7,5
(f) 6
(g) True
(h) True
(i) True
(j) False

Worksheet 1.3

Section 1

1. (a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{9}{19}$ (d) $\frac{45}{61}$ (e) $\frac{128}{5\times325}$

Section 2

1. (a) $\frac{5}{3}$ (b) $\frac{43}{8}$ (c) $\frac{9}{4}$ (d) $\frac{17}{5}$
2. (a) $2\frac{4}{7}$ (b) $2\frac{2}{3}$ (c) $1\frac{2}{3}$ (d) $9\frac{1}{5}$

Section 3

1. (a) $\frac{7}{15}$ (b) $\frac{39}{70}$ (c) $\frac{18}{5}$ (d) $\frac{39}{7}$
2. (a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{7}{37}$ (d) 8
3. (a) $\frac{7}{10}$ (b) $\frac{40}{33}$ (c) $\frac{13}{6}$ (d) $\frac{103}{45}$

Section 4

1. (a) $\frac{26}{21}$
(b) $\frac{7}{24}$
(c) $\frac{43}{10}$
(d) $11\frac{1}{36}$
(e) $7\frac{1}{2}$
(f) $5\frac{7}{9}$
(g) $3\frac{3}{4}$

Exercises 1.3

1. (a) i. $\frac{2}{5}$ ii. $\frac{3}{5}$ iii. $\frac{5}{6}$
   (b) i. 10 ii. 15 iii. 36
   (c) i. $\frac{17}{7}$ ii. $\frac{51}{8}$ iii. $\frac{53}{12}$
   (d) i. $6\frac{2}{9}$ ii. $4\frac{1}{2}$ iii. $9\frac{1}{4}$
   (e) i. 3 ii. $\frac{1}{5}$ iii. $\frac{9}{4}$
   (f) i. $\frac{1}{5}, \frac{3}{5}, \frac{7}{5}$
   ii. $\frac{1}{100}, \frac{1}{20}, \frac{1}{10}, \frac{1}{5}$
   iii. $\frac{3}{8}, \frac{5}{12}, \frac{2}{3}$

2. (a) $\frac{5}{8}$
   (b) $1\frac{7}{12}$
   (c) $\frac{1}{25}$
   (d) $\frac{11}{18}$
   (e) $5\frac{3}{4}$
   (f) $\frac{15}{56}$
   (g) $\frac{4}{5}$
   (h) $\frac{5}{27}$
   (i) $\frac{1}{10}$
   (j) 2
   (k) 92
   (l) $\frac{7}{12}$

3. (a) $\frac{1}{100000}$
   (b) $\frac{1}{86400}$
   (c) $\frac{3}{5}$
   (d) Rowan

Worksheet 1.4

Section 1

1. (a) 0.3
   (b) 0.15
   (c) 0.008
   (d) 0.367
   (e) 8.5
   (f) 0.5
   (g) 0.75
   (h) 0.8
   (i) 0.85
   (j) 0.38

Section 2

1. (a) $\frac{7}{10}$ (b) $\frac{8}{25}$ (c) $\frac{13}{125}$ (d) $\frac{1}{125}$ (e) $\frac{13}{1000}$

Section 3

1. (a) 0.913 (b) 0.479 (c) 0.64 (d) 73.722 (e) 30.879

Exercises 1.4

1. (a) i. 6 tens, 0 ones, 3 tenths, 1 hundredth
   ii. 7 hundreds, 0 tens, 4 ones, 2 tenths
   iii. 1 ten, 4 ones, 2 tenths, 9 hundredths, 6 thousandths

   (b) i. 604.09
   ii. 80.002
   iii. 9.203
   iv. 64.1046
   v. 162.0109
   vi. 1000.39

2. (a) i. $\frac{1}{5}$
   ii. $\frac{1}{25}$
   iii. $\frac{1}{500}$
   iv. $\frac{3}{25}$
   v. $\frac{639}{1000}$
   vi. $1\frac{7}{10}$
   vii. $6\frac{1}{25}$
   viii. $\frac{5}{8}$
   ix. $\frac{3}{10}$

   (b) i. 0.1
   ii. 0.02
   iii. 0.54
   iv. 0.804
   v. 0.15
   vi. 0.24
   vii. 0.75
   viii. 0.125
   ix. 0.2

   (c) i. 0.875 ii. 0.3333 iii. 1.2857

3. (a) 0.25
   (b) 0.28
   (c) 12.82
   (d) 0.893
   (e) 4.344
   (f) 4.53

Worksheet 1.5

Section 1

1. (a) $\frac{4}{7}$
   (b) $\frac{3}{7}$

2. (a) $\frac{12}{25}$ (b) $\frac{13}{25}$ (c) $\frac{4}{25}$ (d) $\frac{21}{25}$

Section 2

1. (a) 90% (b) 20% (c) $37\frac{1}{2}\%$ (d) 60% (e) 85%
2. (a) $\frac{6}{25}$ (b) $\frac{3}{5}$ (c) $\frac{9}{20}$ (d) $\frac{3}{20}$ (e) $\frac{17}{200}$
3. (a) 0.64 (b) 0.08 (c) 0.215 (d) 0.19 (e) 0.024

Section 3

1. (a) 280 mL (b) \$5.2 (c) 1920 mL
2. (a) \$744 (b) \$7560 (c) \$85.5
3. \$525,000
4. \$450

Exercises 1.5

1. (a) i. 87% ii. 60% iii. 75% iv. 80% v. $33\frac{1}{3}\%$ vi. 52.38%
   (b) i. $\frac{1}{5}$ ii. $\frac{33}{50}$ iii. $1\frac{1}{5}$ iv. $\frac{3}{4}$ v. $\frac{27}{200}$ vi. $\frac{3}{50}$
2. (a) \$200 (b) 60 cm (c) \$3.10
3. (a) 20% off is cheaper
   (b) 335 ml
   (c) 6.25%
   (d) 51750 people

Worksheet 1.6

Section 2

1. (a) 2
   (b) -5
   (c) -10
   (d) 4
   (e) -1
   (f) -1
   (g) 0
   (h) -49
   (i) -8
   (j) -10

Exercises 1.6

1. (a) 5
   (b) -5
   (c) -19
   (d) 8
   (e) -1
   (f) 10
   (g) 0
   (h) -20
   (i) -11

2. (a) 13.1
   (b) -2.32
   (c) -516.3
   (d) -26.2
   (e) 79.55
   (f) 51.93

3. (a) $-\$850$, $-\$600$
   (b) -5, 15
   (c) 15 degrees
   (d) -6.3 kg
   (e) 9 km north

Worksheet 1.7

Section 1

1. (a) -12
   (b) -14
   (c) 16
   (d) 32
   (e) 36
   (f) -6
   (g) $\frac{1}{8}$
   (h) 9
   (i) -3.6
   (j) 14.7

Section 2

1. (a) 4
   (b) 4
   (c) 2
   (d) 2
   (e) 9
   (f) -16
   (g) $\frac{5}{2}$
   (h) -128
   (i) $-\frac{4}{7}$
   (j) $-\frac{1}{4}$

Section 3

1. (a) 60
   (b) -36
   (c) -400
   (d) -48
   (e) -30
   (f) 10
   (g) -90
   (h) $-\frac{5}{2}$
   (i) -12
   (j) -240
   (k) -3
   (l) $-\frac{1}{7}$
   (m) $-\frac{11}{10}$
   (n) $\frac{10}{3}$
   (o) 120
   (p) $\frac{8}{3}$
   (q) -105
   (r) 6
   (s) -20
   (t) -8

Exercises 1.7

1. (a) i,iv and v are the same. ii, iii and vi are the same.
   (b) i, ii and iv are equivalent. iii, v and vi are equivalent.
   (c) i. -60 ii. 100 iii. -15 iv. 4 v. -4 vi. 40 vii. -126 viii. 26 ix. -33 x. -11 xi. -24 xii. 2
   (d) i. 4 ii. 5 iii. 1 iv. 0 v. 8 vi. 2

Worksheet 1.8

Section 1

1. (a) 6^{10} (b) 4^7 (c) x^{16} (d) m^7 (e) m^{12} (f) 8^6 (g) 5^{12} (h) x^{21} (i) x^{17} (j) m^{15}

Section 2

1. (a) 6^3 (b) 10^3 (c) x^{10} (d) x^{-6} (e) y^{-7} (f) y^5 (g) 7^6 (h) m^7 (i) y^{15} (j) 8^6

Section 3

1. (a) 3 (b) 3 (c) 4 (d) $\frac{1}{4}$ (e) $\frac{1}{9}$
2. (a) $8^{\frac{1}{2}}$ (b) $m^{\frac{1}{2}}$ (c) m^3 (d) $10^{\frac{3}{2}}$ (e) 16^{-1}

Exercises 1.8

1. (a) $3^3 \times 2^2$ (b) 3^3 (c) i & iv, v & vi (d) 5^{-2} (e) 3^{-3} (f) $64^{\frac{1}{2}}$

2. (a) 128
   (b) 3^{10}
   (c) $\frac{1}{144}$
   (d) 2.3^{-2}
   (e) $\frac{1}{2}$
   (f) 4^{-7}
   (g) 52
   (h) 0.0001
   (i) $10^{11}3^{-6}$

Worksheet 1.9

Section 2

1. (a) $5x$
   (b) $xy + x$
   (c) $2x^2y + x^2 + 2y^2$
   (d) $\frac{1}{2} - \frac{1}{3}x$
   (e) $4 + 7x + y$
   (f) $xyz - 2yz + xz$

Section 3

1. (a) -3 (b) $2x$ (c) y (d) $\frac{7}{4}x^2 - \frac{3}{4}y^2$

2. (a) $2x^2 + 2$
   (b) $(2x + y)(2x + y - 1)$
   (c) $-2xy - y^2$
   (d) $\frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{4}$

Exercises 1.9

1. (a) $7a$
   (b) $3a + 2b$
   (c) $8x^2 - x$
   (d) $x - y$
   (e) $2y - x$
   (f) $2ab + 2a$
   (g) $4x$
   (h) $2x^3 - x^2 - 7$
   (i) $2q$
   (j) $-\frac{1}{3}x - \frac{1}{8}y$

2. (a) $C = 15p$ (b) $C = 0.9b + 1.2r$ (c) $N = n + m$ (d) $t = 4w$

Worksheet 1.10

Section 1

1. (a) $6x + 18$
   (b) $6x - 8$
   (c) $-3x - 3y$
   (d) $5m - 20$
   (e) $x^2 + xy$
   (f) $y^3 - 2y$
   (g) $3t^2 + 4t$
   (h) $4y - 20$
   (i) $-x^2 - 2x$
   (j) $-3m + 3n$

2. (a) $5m + 26$
   (b) $t - 11$
   (c) $5m - 13$
   (d) $-x + 2$
   (e) $13t + 11$
   (f) $2x - 2$
   (g) $2x^2 + x$
   (h) $t^2 - 3t - 8$
   (i) $m^2 - 2m + 2$
   (j) $7x + 6$

Section 2

1. (a) $x^2 + 7x + 10$
   (b) $x^2 + 5x + 4$
   (c) $y^2 - y - 6$
   (d) $m^2 + 2m - 35$
   (e) $x^2 + 5x - 24$
   (f) $2x^2 + 5x + 2$
   (g) $3m^2 - 10m - 8$
   (h) $x^2 - 9$
   (i) $y^2 - 25$
   (j) $m^2 + 10m + 25$

2. (a) $9x + 11$ (b) $9x + 6$ (c) $14x + 28$ (d) $3m + 10$ (e) $-17m - 27$

Section 3

1. (a) 8
   (b) -3
   (c) 13
   (d) 16
   (e) 2
   (f) 66

2. (a) 8
   (b) 36
   (c) 33
   (d) 4
   (e) 14
   (f) 34
   (g) 14
   (h) 56
   (i) 49
   (j) 9

Exercises 1.10

1. (a) $3p + 3q$
   (b) $3a^2 + 12a$
   (c) $-4ab + 24a$
   (d) $3x - 6$
   (e) $3x + 7y$
   (f) $2x^2 + 5x + 3$
   (g) $x^2 + x - 6$
   (h) $6x^2 - 31x + 18$
   (i) $6x - 8y + 11$
   (j) $2x^3 + 11x^2 + 17x + 6$

2. (a) 12
   (b) 40
   (c) 44
   (d) $s = 20$
   (e) $c = 22.7$ grams
   (f) 33